Hochschule
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Institute for AI and
Autonomous Systems

Relational Learning

A Short Introduction

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Structure

- Relational learning preliminaries
- Relational learning methods

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A hybrid skill parameterisation model combining symbolic and subsymbolic elements for introspective robots

Alex Mitrevski [a,b,*], Paul G. Plöger [a], Gerhard Lakemeyer [b]

Alex Mitrevski

Improving the Manipulation Skills of Service Robots by Refining Action Representations

Skill generalisation and experience acquisition for predicting and avoiding execution failures = Fähigkeitsgeneralisierung und Erfahrungserwerb zum Vorhersagen und Vermeiden von Ausführungsfehlern

Mitrevski, Aleksandar (Alex)

Relational Learning Preliminaries

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- In our first lectures, we defined a skill as $\mathcal{S} = (\mathcal{S}_{\mathcal{I}}, \mathcal{S}_{\mathcal{T}}, \pi)$, where $\mathcal{S}_{\mathcal{I}}$ are skill preconditions and $\mathcal{S}_{\mathcal{T}}$ are skill termination conditions, but we did not discuss how those can be acquired — this is our general objective for today

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- Beyond skill models that can be used for autonomous planning, relational learning has another use: it acquires **explainable models of execution**

Relational learning is a problem of extracting qualitative rules from observations, or learning models of relations themselves

Relational Learning and Planning Domains

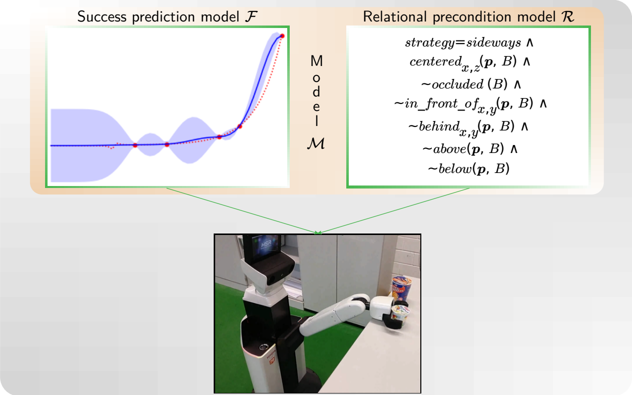
- A traditional use of relational learning is for **learning planning domains from observed plan traces** so that there is no need for writing domains manually

The snippet is based on an example used in the presentation of A. Mitrevski et al., "On the Diagnosability of Actions Performed by Contemporary Robotic Systems," in *31th Int. Workshop Principles of Diagnosis (DX)*, 2020.

```
(:action Pick
    :parameters (?Object - Object ?Table - Table ?Robot - Robot ?Waypoint -
           Waypoint)
    :precondition (and
         (robotAt ?Robot ?Waypoint)
         (tableAt ?Table ?Waypoint)
         (on ?Object ?Table)
         (emptyGripper ?Robot)
    )
    :effect (and
         (not (on ?Object ?Table))
         (not (emptyGripper ?Robot))
         (holding ?Robot ?Object)
    )
)
```

Relational Learning and Execution

- Relational models can also be used to describe execution rules, but specifying those manually is not a generalisable approach; **relational learning can also be used for learning execution models**



A. Mitrevski, P. G. Plöger, and G. Lakemeyer, "A Hybrid Skill Parameterisation Model Combining Symbolic and Subsymbolic Elements for Introspective Robots," *Robotics and Autonomous Systems*, vol. 161, p. 104350:1–22, Mar. 2023.

Relational Learning Tasks

Skill precondition learning

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Symbol learning

The other three tasks assume that a set of relevant relations is given and these are used to learn appropriate rules; in symbol learning, the objective is to **learn the relations themselves**

Relational Learning Methods

Type of Relational Learning Tasks

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If the relations are unknown, experiences need to be used to find new relations; this is a **symbol learning task**

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Unknown relations

If the relations are unknown, experiences need to be used to find new relations; this is a **symbol learning task**

- In this lecture, we will mostly focus on the case of learning with known relations as it is a better studied problem; symbol learning will be discussed only briefly

Learning Objective: Known Relations

- Let us suppose that we have **a set P of candidate relations**, where $r \in P$ is defined as either $r : S \rightarrow \mathbb{Z}$ or $r : S \rightarrow \mathbb{R}$, with S a state representation
  - E.g. P can be a set such as $\{leftOf(X, Y), rightOf(X, Y), inFrontOf(X, Y), behind(X, Y)\}$
  - Typically, the relations we care about are logical, so they only take two values — $true$ and $false$ — but we consider a general set of qualitative and continuous relations here

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- The learning objective is to **identify a subset $\mathcal{R} \subseteq P$** so that $\mathcal{R}$ is **consistent with a set of training examples**
  - If P only has logical relations, we can interpret $\mathcal{R}$ as $\bigwedge_r r, r \in \mathcal{R}$

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- Particularly in the context of learning planning domains, we are usually given **a set of plan traces Π**, where each $\pi \in \Pi$ is a sequence of the form $(\mathcal{R}_i, \mathcal{A}_i, \mathcal{E}_i)$, where:
  - $\mathcal{A}_i$ is an action
  - $\mathcal{R}_i$ are the observed preconditions when executing $\mathcal{A}_i$
  - $\mathcal{E}_i$ are the observed effects after executing $\mathcal{A}_i$

Learning Data

- For learning, we consider **ordered vectors** $\boldsymbol{z}_j, 1 \leq j \leq n$, where each element is an instantiated (i.e. ground) relation
  - E.g. considering the previous example set P and the relations defined for objects $X = cup$ and $Y = bottle$, we may have $\boldsymbol{z} = (1, 0, 0, 1)^T$, which we interpret as

$$leftOf(cup, bottle) \wedge \neg rightOf(cup, bottle) \wedge \neg inFrontOf(cup, bottle) \wedge behind(cup, bottle)$$

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- If all experiences are given as a batch, we consider:
  - **a matrix of** m **examples** $Z \in \mathbb{Z}^{m \times n}$, where each row $Z_i, 1 \leq i \leq m$ is an ordered vector as above
  - **a vector of** m **labels** $\boldsymbol{y}$, where each $\boldsymbol{y}_i \in \{0, 1\}, 1 \leq i \leq m$ denotes whether an experience is positive (1) or negative (0)

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- We can, for convenience, split the positive and negative experiences in Z into
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  - **Z^+, a matrix of positive examples**, namely examples where $\boldsymbol{y} = 1$
  - **Z^-, a matrix of negative examples**, namely examples where $\boldsymbol{y} = 0$
- We will also define $r^{-1}(\boldsymbol{z})$, which is an operator that returns a lifted version of r
  - E.g. $leftOf(cup, bottle)$ is ground, while $leftOf(X, Y)$ is lifted

Overview of Relational Learning Methods

There is a large variety of relational learning methods that can roughly be categorised as follows:

Hypothesis search

Relational learning is observed as a heuristic-driven search problem that looks for rules that describe the plan traces Π, for instance using inductive logic programming

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Relational reinforcement learning

A variant of reinforcement learning (traditionally of Q-learning), where the state-action value function is learned for relational states and actions

Online Precondition Learning

- We will consider an online learning procedure[1] defined for logical relations that **updates a relation set with each new experience**

[1] X. Wang, "Learning Planning Operators by Observation and Practice," in *Proc. 2nd Int. Conf. Artificial Intelligence Planning Systems*, pp. 1994, 335–341.

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- The procedure starts with an empty set of relations $\mathcal{R}_0 = \{\}$ and, when the first **positive** execution is observed, **all relations that hold are added to $\mathcal{R}_1$**:

$$\mathcal{R}_1 = \{r^{-1}(\boldsymbol{z}_j) \mid \boldsymbol{z}_j = 1\}, 1 \leq j \leq n$$

  In subsequent experiences, the update depends on the value of y

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- In the case of a positive experience, **predicates that do not hold are removed from the set**:

$$\mathcal{R}_t = \mathcal{R}_{t-1} \setminus \{r^{-1}(\boldsymbol{z}_j) \mid r^{-1}(\boldsymbol{z}_j) \in \mathcal{R}_{t-1} \wedge \boldsymbol{z}_j = 0\}, 1 \leq j \leq n$$

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- In the case of a negative experience in which $\mathcal{R}_{t-1}$ holds before the execution, **the set is expanded by the negations of predicates that hold in z, but which are not in $\mathcal{R}_{t-1}$**:

$$\mathcal{R}_t = \mathcal{R}_{t-1} \cup \{\neg r^{-1}(\boldsymbol{z}_j) \mid r^{-1}(\boldsymbol{z}_j) \notin \mathcal{R}_{t-1} \wedge \boldsymbol{z}_j = 1\}, 1 \leq j \leq n$$

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Online Effect Learning

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$$\boldsymbol{\Delta} = \boldsymbol{z}_t - \boldsymbol{z}_{t-1}$$

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- As in the case of the preconditions, the effect set is initialised with an empty set $\mathcal{E}_0 = \{\}$, and **all effects that are observed in the delta state are added to $\mathcal{E}$ after the first positive execution**:

$$\mathcal{E}_1 = \{r^{-1}(\boldsymbol{\Delta}_j) \mid \boldsymbol{\Delta}_j \neq 0\}, 1 \leq j \leq n$$

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- The effects are only updated in the case of a positive experience, such that **predicates that are part of the delta state are included in $\mathcal{E}$**:

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At $t = 2$, we observe another positive experience:

$$\{leftOf(cup, bottle) = true, inFrontOf(X, Y) = false, red(cup) = false, blue(bottle) = false\}$$

$$\implies \boldsymbol{z} = (1, 0, 0, 0)$$

$$\implies \mathcal{R}_2 = \{leftOf(X, Y)\}$$

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At $t = 3$, we observe a negative experience:

$$\{leftOf(cup, bottle) = true, inFrontOf(X, Y) = true, red(cup) = true, blue(bottle) = false\}$$

$$\implies \boldsymbol{z} = (1, 1, 1, 0)$$

$$\implies \mathcal{R}_3 = \{leftOf(X, Y), \neg blue(Y)\}$$

Statistical Learning: Qualitative Relations

- In this context, we will consider at a procedure that, given Z^+, looks for relations whose values have **little variation between executions**[2]

[2] N. Abdo et al., "Learning Manipulation Actions From a Few Demonstrations," in *Proc. IEEE Int. Conf. Robotics and Automation (ICRA)*, May 2013, pp. 1268–1275.

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- For a qualitative relation r_j, the variation between experiences over Z^+ is computed by the **entropy**:
$$J(Z_j^+) = -\sum_v P(Z_j^+ = r_j^v) \log P(Z_j^+ = r_j^v)$$
and r_j is taken to be a relevant precondition or effect if $J(Z_j^+) < \delta$ for a predefined threshold δ

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  and r_j is taken to be a relevant precondition or effect if $J(Z_j^+) < \delta$ for a predefined threshold δ
- The value r_j^* of r_j that is taken as a precondition or effect is then

$$r_j^* = \arg\max_v P(Z_j^+ = r_j^v)$$

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- Concretely, let us consider a continuous feature f_j whose values are combined in a matrix F_j; the **values in F are then clustered into K clusters** and J is defined as the **average distance to the cluster centroids**:

$$J(F_j^+) = \frac{1}{KN} \sum_{k=1}^{K} \sum_{i=1}^{N_k} d\left(F_{j_i}^{k+}, \boldsymbol{\mu}_k\right)$$

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- Concretely, let us consider a continuous feature f_j whose values are combined in a matrix F_j; the **values in F are then clustered into K clusters** and J is defined as the **average distance to the cluster centroids**:

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  where $F_{j_i}^{k+}, 1 \leq i \leq N_k$ are points in the positive executions that are assigned to cluster k and d is the Euclidean distance
- As in the case of qualitative relations, **f_j is taken to be a precondition or effect if $J(F_j^+) < \delta$**
- During online use, **the feature vector $\boldsymbol{f}$ is assigned to a cluster**, and **the relation is considered to be satisfied if $d(\boldsymbol{f}, \boldsymbol{\mu}_c) < d_{\max}$**, where $\boldsymbol{\mu}_c$ is the centroid of the cluster and $d_{\max}$ is a predefined distance threshold

Statistical Learning Example: Qualitative Relations

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Let us assume that we obtain the following observations in positive executions for $X = cup$ and $Y = bottle$:

$$Z^{+T} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 3 & 3 & 2 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 3 & 1 \end{pmatrix}^T$$

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We then have the following entropy values (with $\ln$):

$leftOf(X,Y)$	$inFrontOf(X,Y)$	$color(X)$	$color(Y)$
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If we take $\delta = 0.2$, then $\mathcal{R} = leftOf(cup, bottle)$

Learning as Hypothesis Search

- In the hypothesis context search, we will consider a technique[3] that **first learns delta rule predictive models and then looks for general explicit rules that describe the examples in Π**

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- Given Π and vectors $\boldsymbol{z}^{pre}$ and $\boldsymbol{z}^{post}$ that describe the state before and after executing an action, respectively, **a voted perceptron model for each action that predict a delta state**, namely each model is of the form $p_{\mathcal{A}_i}(\boldsymbol{z}) = \Delta$
  - For training the voted perceptron, a binary kernel is used, which has the form $k(\boldsymbol{x}, \boldsymbol{y}) = 2^{\sum_i \mathbb{1}_{x_i = y_i}}$, where $*$ values are not considered

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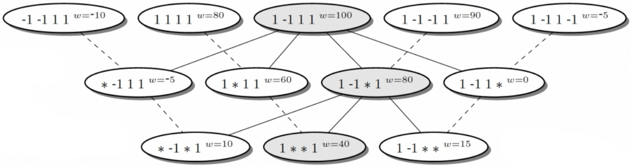
  - **A precondition set is extracted for each effect**, where the extraction process looks for **the most general rule that predicts the effect** (i.e. a precondition that has as many unknown relation values as possible), where the perceptron weighs the vectors



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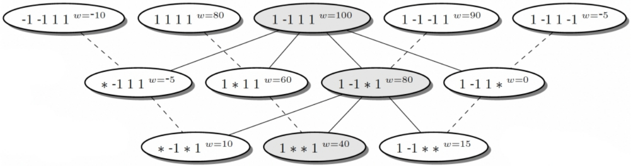

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  - **The rules for the individual effects are merged** in an iterative process that combines preconditions / effects by ensuring that there are no conflicts in the rules, and then attempts to simplify the rules by generalising the relations

Optimisation-Based Learning

Input: a set of plan traces: $T^o = \{t^o\}$.
Output: a domain model m^*.

initialize $\vec{\theta}$ and $\vec{w}$ with random values, and set an iteration number R;
for $r = 1$ to R **do**
  for each $t^o = \langle s_0, a_1^o, \ldots, a_n^o, g \rangle \in T$ **do**
    sample t and m based on $p(t, m|t^o)$;
    calculate the reward $r(t, m)$ based on Equation (7);
    update $\vec{\theta}$ and $\vec{w}$ based on Equations (12) and (13);
  end for
end for
$m^* = \arg\max_{m \in M} w_m$;
return m^*;

An example of an optimisation-based procedure for learning a planning domain. Algorithm from H. H. Zhuo and S. Kambhampati, "Action-model Acquisition from Noisy Plan Traces," in *Proc. 23rd Int. Joint Conf. Artificial Intelligence (IJCAI)*, 2013, pp. 2444–2450.

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- This can be done by **defining a parameterised distribution over models** and an associated parameterised optimisation objective, for instance evaluating successful executions under a given model
- If a gradient-based optimisation procedure is used, this type of relational learning can also be performed in the context of neural networks

Relational Reinforcement Learning

Initialize $\hat{Q}_0$ to assign 0 to all (s, a) pairs
Initialize Examples to the empty set.
e := 0
do forever
  e := e + 1
  i := 0
  generate a random state s_0
  while not goal(s_i) **do**
    select an action a_i stochastically
      using the Q-exploration strategy from Equation (1)
      using the current hypothesis for $\hat{Q}_e$
    perform action a_i
    receive an immediate reward $r_i = r(s_i, a_i)$
    observe the new state s_{i+1}
    i:=i+1
  endwhile
  for j=i−1 to 0 **do**
    generate example $x = (s_j, a_j, \hat{q}_j)$,
      where $\hat{q}_j := r_j + \gamma max_{a'} \hat{Q}_e(s_{j+1}, a')$
    if an example $(s_j, a_j, \hat{q}_{old})$ exists in Examples, replace it with x,
      else add x to Examples
  update $\hat{Q}_e$ using TILDE-RT to produce $\hat{Q}_{e+1}$ using Examples

S. Dzeroski, L. De Raedt, and H. Blockeel, "Relational reinforcement learning," *Proc. 8th Int. Conf. Inductive Logic Programming (ILP)*, 1998.

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- The difference with traditional reinforcement learning is in the Q-function representation — in RRL, **a logical decision tree is used to model the function**, where each node in the tree is **a binary decision node**
- As is typical in reinforcement learning, the Q-function is continuously updated; in RRL, the update is performed **after finding a sequence of actions that lead to a desired goal**
  - **The tree is induced from a set of examples that is regularly updated** (much like an experience buffer)

Symbol Learning

- For all previous approaches, we made **an assumption that the set of relations P from which we extract relational rules is given**, but this puts a manual design burden
  - The manual definition of relations is challenging particularly because we cannot always guarantee that the used set contains all **relevant relations** for describing a problem

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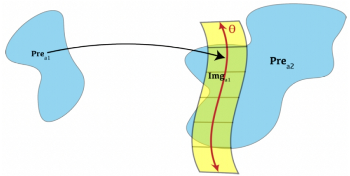

B. Ames, A. Thackston and G. Konidaris, "Learning Symbolic Representations for Planning with Parameterized Skills," in *Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems (IROS)*, 2018, pp. 526–533.

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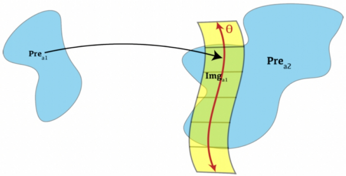

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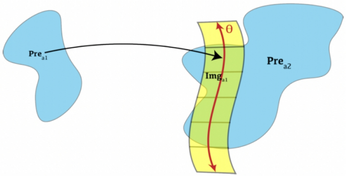

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- There is no general solution to the symbol learning problem — the problem is too unconstrained — but there are existing approaches for it in the context of **parameterised skills**
- In this context, **symbols are learned in terms of the skill parameters**, for instance using clustering, and can be seen as **discretisations of the parameter space**
  - The learned symbols enable transitions between the created clusters

Summary

- Relational learning is a problem of extracting relational rules from observations, typically plan traces
- Relational learning can be used to learn skill models (namely skill initiation and termination conditions), but also to learn execution rules
- There are various relational learning methods, which we categorised into hypothesis search, online learning, statistical learning, optimisation-based, and relational reinforcement learning
- Symbol learning is the problem of acquiring models of relations, thereby eliminating the need for manual specification of relations