Hochschule
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University of Applied Sciences

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Institute for AI and Autonomous Systems

Learning from Demonstration

An Overview With a Focus on Trajectory Learning

Dr. Alex Mitrevski
Master of Autonomous Systems

Structure

- What is learning from demonstration?
- Learning from one demonstration: Dynamic motion primitives
- Learning from multiple demonstrations: Gaussian mixture models

Annual Review of Control, Robotics, and Autonomous Systems

Recent Advances in Robot Learning from Demonstration

Harish Ravichandar,[1,*] Athanasios S. Polydoros,[2,*] Sonia Chernova,[1,†] and Aude Billard[2,†]

Dynamical Movement Primitives: Learning Attractor Models for Motor Behaviors

In Special Collection: CogNet

Auke Jan Ijspeert, Jun Nakanishi, Heiko Hoffmann, Peter Pastor, Stefan Schaal

Neural Computation (2013) 25 (2): 328–373.

IEEE TRANSACTIONS ON ROBOTICS, VOL. 27, NO. 5, OCTOBER 2011 943

Learning Stable Nonlinear Dynamical Systems With Gaussian Mixture Models

S. Mohammad Khansari-Zadeh and Aude Billard

Learning from Demonstration Preliminaries

What is Learning from Demonstration?

F. J. Abu-Dakka et al. "Solving peg-in-hole tasks by human demonstration and exception strategies," *Industrial Robot: An International Journal*, vol. 41, no. 6, pp. 575–584, 2014.

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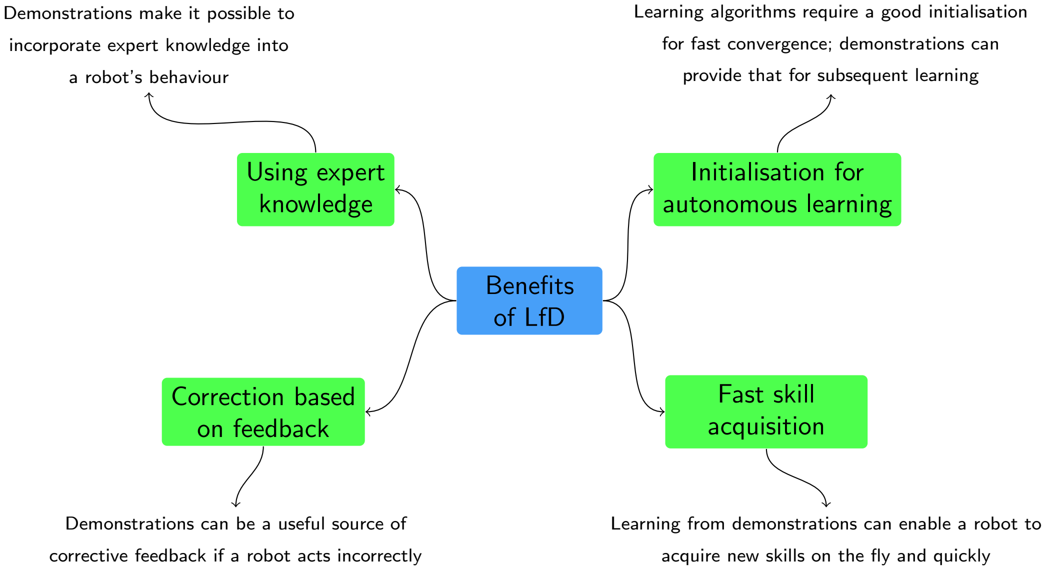
- This can essentially be seen as **a robot programming technique without writing explicit programs** — the knowledge about the robot's behaviour comes from the demonstrations rather than through manually written code
- Demonstration-based learning can **enable end users to teach skills to a robot** without expert knowledge in robotics

Benefits of Learning from Demonstration



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The correspondence problem is concerned with how observations of one embodiment can be mapped to another — potentially different — embodiment

Comparison of the Demonstration Types

Kinesthetic teaching

Advantages

- Demonstrations are quite simple to perform
- Learning is simplified — the correspondence problem is eliminated

Disadvantages

- Not all robots have an interface that supports kinesthetic teaching
- Primarily useful for manipulator motions

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- More flexible demonstration interfaces
- Can be more easily extended to more robot embodiments

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- The demonstration is not necessarily intuitive
- Specialised hardware may be necessary for performing demonstrations

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Passive observation

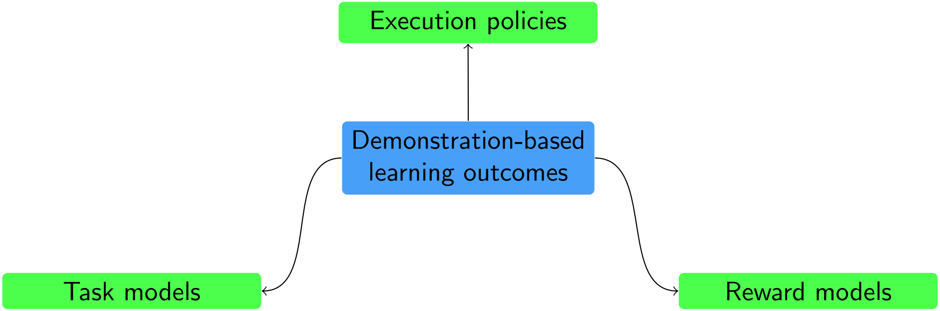
Advantages

- Simplest to perform for demonstrators
- Applicable to all types of robots

Disadvantages

- Challenging information extraction
- The learning is more complicated — the correspondence problem needs to be solved

What Can Be Learned from Demonstrations?



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  - **it may not always be clear how to combine information from multiple demonstrations** (particularly if the demonstrations show seemingly contradictory behaviour)

Segmentation of Demonstrations

- In the case of learning based on passive observations, **demonstrations may contain sequences of actions of interest**
- In such cases, it is required to **perform demonstration segmentation** so that individual actions of interest can be extracted and then learned
- This is **particularly the case when learning full task models from demonstrations**
- **Changing contact interactions with objects** represent one common criterion that is used for segmenting observations
  - NB: These are equivalent to the mode transitions that we discussed in the lecture on learning for manipulation!

Learning Trajectories from One Demonstration: Dynamic Motion Primitives

(Reminder) Homogeneous Second-Order Differential Equations

- Consider the general form of a **homogeneous, second-order differential equation with constant coefficients**:

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Complex roots $k_{1/2} = \lambda \pm \mu i$ (underdamped system)

$$y(t) = c_1 e^{\lambda t} \cos(\mu t) + c_2 e^{\lambda t} \sin(\mu t)$$

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where $f(t)$ is an **external forcing term**

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- The general solution of such an equation has the form

$$y(t) = y_c(t) + y_p(t)$$

  where
  - $y_c(t)$ **is a complementary solution** (the solution to the homogeneous equation) and
  - $y_p(t)$ **is a particular solution** (a specific solution to the nonhomogeneous equation)

(Reminder) Autonomous Differential Equation

- A dynamical system of equations that does not explicitly depend on the input variable is called an **autonomous system** and has the form

$$\frac{dy}{dt} = f(y(t))$$

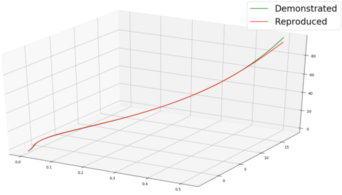
- A **non-autonomous system** is one where the dependence on the input variable is explicit:

$$\frac{dy}{dt} = f(y(t), t)$$

- If the independent variable of the system represents time, an autonomous system is also called **time-invariant**

Dynamic Motion Primitive (DMP) Idea

- In the DMP framework, **a trajectory is modelled as a second-order dynamical system with an external forcing term**
- The system is defined so that **a desired goal can be reached**
- The external forcing term **imposes a given shape on the overall trajectory**
- Learning a trajectory from demonstration is achieved by **parameterising the forcing term and learning its parameters based on a demonstration**



An example trajectory and its reproduction using with a learned DMP

System Formulation

- Formally, a DMP is defined as

$$\tau \ddot{y} = \alpha \left(\beta \left(g - y \right) - \dot{y} \right) + f$$

Here:

- y is the robot's state (e.g. position)
- g is a desired goal
- f is a forcing term
- τ is a time constant (controls the trajectory's speed)
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- The system will converge to g if it is critically damped and $f \to 0$
  - Critical damping can be achieved by setting $\beta = \alpha/4$

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- This is often rewritten in a first-order form with $z = \tau\dot{y}$ and

$$\tau\dot{z} = \alpha_z\left(\beta_z(g - y) - z\right) + f$$

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Forcing Term

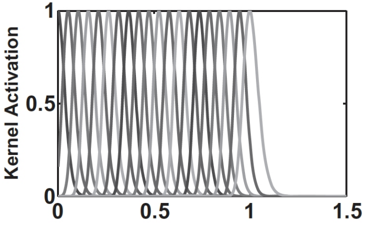
- The DMP forcing term is represented as **a linear combination of basis functions**:

$$f(t) = \frac{\sum_{i=1}^{n} \Psi_i(t) w_i}{\sum_{i=1}^{n} \Psi_i(t)}$$

- Each basis function is **a Gaussian kernel** of the form

$$\Psi_i(x) = \exp\left(-\frac{1}{2\sigma_i}(x - c_i)^2\right)$$

- Learning a DMP amounts to **learning the weights of the forcing term**



Discrete Canonical System

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- As x exponentially decays, the modulation by x ensures that $f \rightarrow 0$
- The term $g - y_0$ achieves spatial scaling depending on the initial distance to the goal

Using DMPs With Multiple Degrees of Freedom

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- Another alternative is to **couple multiple canonical systems**
  - Can be used for synchronising the motion of multiple manipulators

Canonical System for Rhythmic Motions

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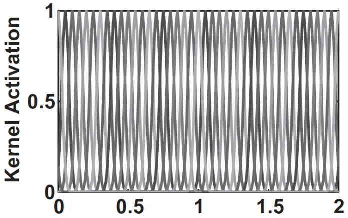
- The basis functions in this case are sinusoidal:

$$\Psi_i(\phi) = \exp\left(h_i\left(\cos\left(\phi - c_i\right) - 1\right)\right)$$



Weight Learning Using Locally Weighted Regression (1/2)

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- A demonstration is **a sequence of T measurements $D = (d_1, ..., d_T)$, where** $d_t = (t, y_{demo_t}, \dot{y}_{demo_t}, \ddot{y}_{demo_t}), 1 \leq t \leq T$
- Substituting the demonstration points into the equation of the system results in expression for the forcing term:

$$\tau^2 \ddot{y}_{demo} - \alpha_z \left(\beta_z \left(g - y_{demo} \right) - \tau \dot{y}_{demo} \right) = f^*$$

- The learning objective is that of **finding weights that bring the DMP forcing term f as close as possible to the desired forcing term f^***; this objective is expressed as

$$J_i = \sum_{i=1}^{T} \Psi_i(t) \left(f^*(t) - w_i \xi(t) \right)^2$$

- In this equation, $\xi(t) = x(g - y_0)$ for a discrete system and $\xi(t) = r$ for a rhythmic system

Dynamical Movement Primitives: Learning Attractor Models for Motor Behaviors
In Special Collection: CogNet
Auke Jan Ijspeert, Jun Nakanishi, Heiko Hoffmann, Peter Pastor, Stefan Schaal
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Weight Learning Using Locally Weighted Regression (2/2)

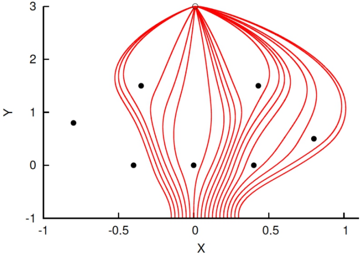
- A solution for the objective can be found using **locally weighted regression**
- Concretely, the weights $w_i, 1 \leq i \leq N$ are found as

$$w_i = \frac{\boldsymbol{\xi}^T \Gamma_i \boldsymbol{f}^*}{\boldsymbol{\xi}^T \Gamma_i \boldsymbol{\xi}}$$

where

$$\boldsymbol{\xi} = \begin{pmatrix} \xi(1) \\ \vdots \\ \xi(t) \\ \vdots \\ \xi(T) \end{pmatrix} \qquad \Gamma_i = \begin{pmatrix} \Psi_i(1) & & & 0 \\ & \ddots & & \\ & & \Psi(t) & \\ & & & \ddots \\ 0 & & & \Psi_i(T) \end{pmatrix} \qquad \boldsymbol{f}^* = \begin{pmatrix} f^*(1) \\ \vdots \\ f^*(t) \\ \vdots \\ f^*(T) \end{pmatrix}$$

Obstacle Avoidance



H. Hoffmann et al., "Biologically-inspired dynamical systems for movement generation: Automatic real-time goal adaptation and obstacle avoidance," in *Proc. IEEE Int. Conf. Robotics and Automation (ICRA)*, 2009, pp. 2587–2592.

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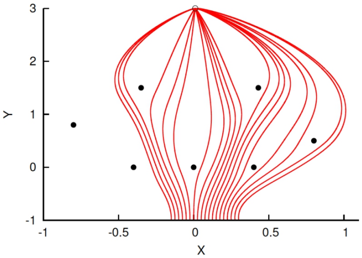

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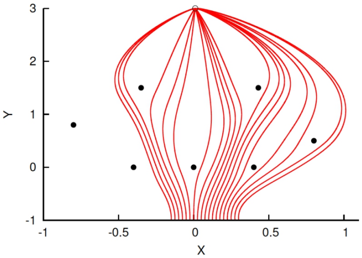

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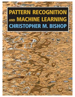
- The dynamic system formulation makes it relatively simple to incorporate additional inputs to the system
- One way in which this can be achieved is by **changing the system equations online to include additional forcing terms** that can change the system's behaviour
- For obstacle avoidance, **the external force should decay exponentially with the distance to the obstacle** so that the system can eventually return to its original goal
  - This idea is similar to how potential fields work

Learning Trajectories from Multiple Demonstrations: Gaussian Mixture Models

(Reminder) Gaussian Mixture Model (GMM)



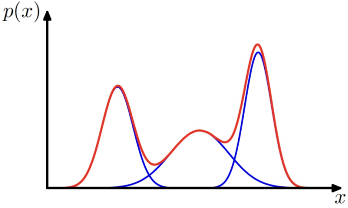
- A multimodal distribution can modelled by **a linear combination of K Gaussian distributions**

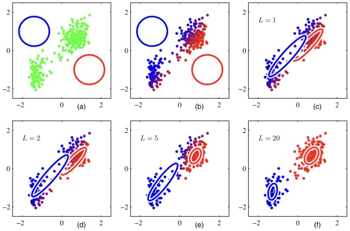
- A Gaussian mixture model (GMM) has the form

$$p(\boldsymbol{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}_k, \Sigma_k)$$

- The parameters π_k are the **mixing coefficients**, such that $0 \leq \pi_k \leq 1$ and $\sum_{k=1}^{K} \pi_k = 1$

- Given data to be modelled by a GMM, the **expectation-maximisation** algorithm is used to find the parameters of the mixture components





Learning from Multiple Demonstrations Objective

- When learning from multiple demonstrations, we are given a **collection of M demonstrations** $\mathrm{T} = \{D_1, ..., D_M\}$
  - Each $D_i, 1 \leq i \leq M$ can be defined as $D_i = (t, \boldsymbol{x}_t, \dot{\boldsymbol{x}}_t), 1 \leq t \leq T$

Learning from Multiple Demonstrations Objective


IEEE TRANSACTIONS ON ROBOTICS, VOL. 27, NO. 5, OCTOBER 2011 943

Learning Stable Nonlinear Dynamical Systems With Gaussian Mixture Models

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- The learning objective is to find a model that:
  - **summarises the distribution of the M demonstrations**
  - **makes it possible to sample trajectories for execution**
- Such a model can be represented by a GMM, and the learning objective becomes that of learning the GMM parameters

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Trajectory GMM

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$$P(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \sum_{k=1}^{K} \pi_k P(\boldsymbol{x}, \dot{\boldsymbol{x}}|k) = \sum_{k=1}^{K} \pi_k P\left(\boldsymbol{x}^{t,i}, \dot{\boldsymbol{x}}^{t,i}|k\right), \; 1 \leq i \leq M, \; 1 \leq t \leq T$$

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- As the components of the mixture are Gaussian distributions, their parameters are the mean and covariance matrix, which are represented as

$$\boldsymbol{\mu}^k = \begin{pmatrix} \boldsymbol{\mu}^k_{\boldsymbol{x}} \\ \boldsymbol{\mu}^k_{\dot{\boldsymbol{x}}} \end{pmatrix} \qquad \Sigma^k = \begin{pmatrix} \Sigma^k_{\boldsymbol{x}} & \Sigma^k_{\boldsymbol{x}\dot{\boldsymbol{x}}} \\ \Sigma^k_{\dot{\boldsymbol{x}}\boldsymbol{x}} & \Sigma^k_{\dot{\boldsymbol{x}}} \end{pmatrix}$$

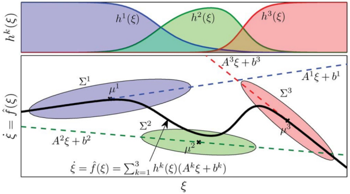
Trajectory Execution

- During execution, **appropriate velocities $\dot{x}$ need to be calculated** so that a robot can execute **a trajectory based on the model**
- The expectation of the posterior estimate $P(\dot{\boldsymbol{x}}|\boldsymbol{x})$ can be found as

$$\dot{\boldsymbol{x}} = \sum_{k=1}^{K} \frac{\pi_k P(\boldsymbol{x}|k)}{\sum_{i=1}^{K} \pi_i P(\boldsymbol{x}|k)} \left(\boldsymbol{\mu}_{\dot{\boldsymbol{x}}}^k + \Sigma_{\boldsymbol{x}\dot{\boldsymbol{x}}}^k \left(\Sigma_{\boldsymbol{x}}^k \right)^{-1} \left(\boldsymbol{x} - \boldsymbol{\mu}_{\boldsymbol{x}}^k \right) \right)$$
$$= \sum_{k=1}^{K} h^k(\boldsymbol{x}) \left(A^k \boldsymbol{x} + \boldsymbol{b}^k \right)$$

where

$$h^k(\boldsymbol{x}) = \frac{\pi_k P(\boldsymbol{x}|k)}{\sum_{i=1}^{K} \pi_i P(\boldsymbol{x}|k)} \quad A^k = \Sigma_{\boldsymbol{x}\dot{\boldsymbol{x}}}^k \left(\Sigma_{\boldsymbol{x}}^k \right)^{-1} \quad \boldsymbol{b}^k = \boldsymbol{\mu}_{\dot{\boldsymbol{x}}}^k - A^k \boldsymbol{\mu}_{\boldsymbol{x}}^k$$



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- Learning from demonstration has various uses, namely it can be used to learn execution policies, complete task models, or reward models
- In the dynamic motion primitives (DMP) framework, motion is modelled by a second-order dynamical system; this system has a force term whose parameters are learned given a demonstration
- Multiple demonstrations can be used to learn a probabilistic model (a trajectory envelope), for instance in the form of a Gaussian mixture model