



Hochschule  
Bonn-Rhein-Sieg  
University of Applied Sciences



# Path Planning

## How a Robot Finds Its Way Around

Dr. Alex Mitrevski  
Master of Autonomous Systems

# Structure

- ▶ Path planning preliminaries
- ▶ Path planning algorithms
- ▶ Local obstacle avoidance

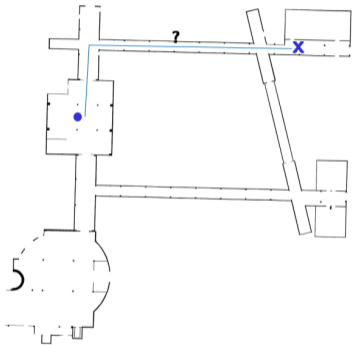


# Path Planning Preliminaries

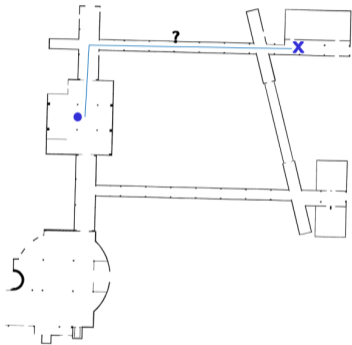


# What is Path Planning?

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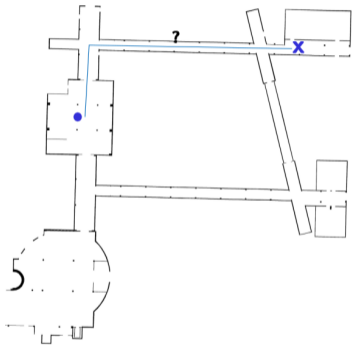


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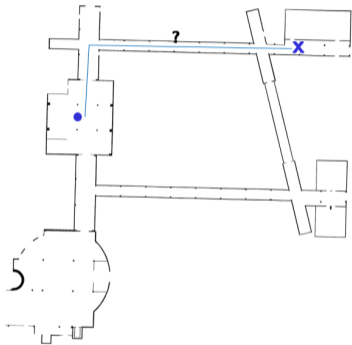
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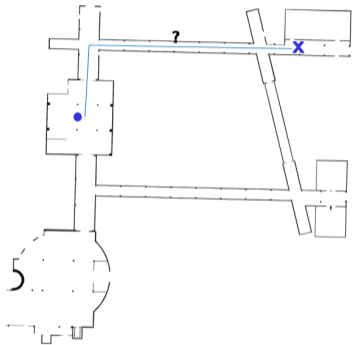
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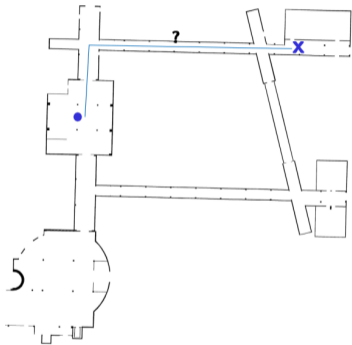
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  2. **How to ensure that  $\mathcal{P}$  does not bring a robot into collisions?**
- ▶ Note that path planning **requires an environment map to be performed** — obstacles need to be known so that collisions with them can be avoided

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- ▶ A desirable property of path planning algorithms that **cannot always be guaranteed** — due to the complexity of a planning configuration, **it may be impossible to find a path within a given time or memory budget**

# Configuration Space

- ▶ Path planning needs to take into account the fact that a robot is not a point in space, but a full body
  - ▶ Achieved by planning not in the robot's physical space, but **in configuration space**, where each configuration is a point



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$$\mathcal{C}_{obs} = \{\mathbf{q} \in \mathcal{C} \mid \mathcal{A}(\mathbf{q}) \cap \mathcal{O} \neq \emptyset\}$$





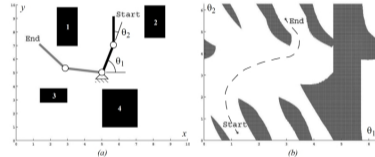
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- ▶ **Obstacles are typically enlarged in the C-space**, and a valid path is one that **passes only through the free space**

$$C_{free} = \mathcal{C} \setminus C_{obs}$$

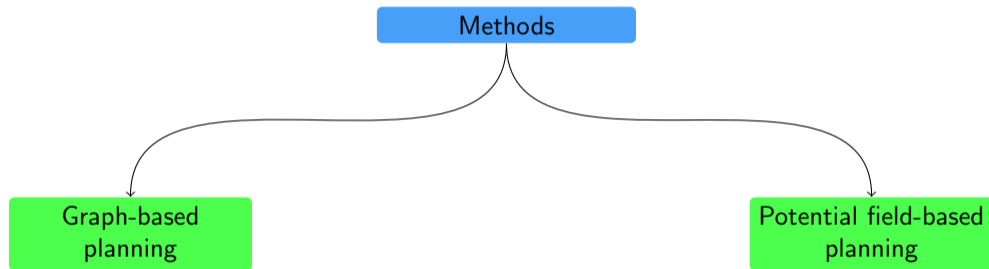


**Figure 6.1** Physical space (a) and configuration space (b): (a) A two-link planar robot arm has to move from the configuration *start* to *end*. The motion is thereby constraint by the obstacles 1 to 4. (b) The corresponding configuration space shows the free space in joint coordinates (angle  $\theta_1$  and  $\theta_2$ ) and a path that achieves the goal.

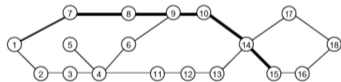
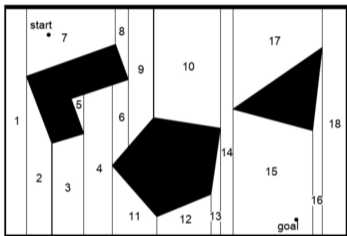
# Path Planning Algorithms



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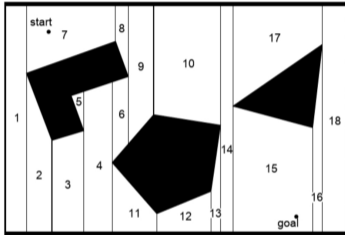


# Graph Search

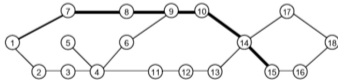
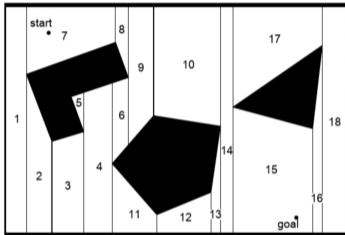


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- ▶ The decompositions that we looked at in the last lecture (e.g. the exact cell decomposition) can be used as precursors to path planning using graph search

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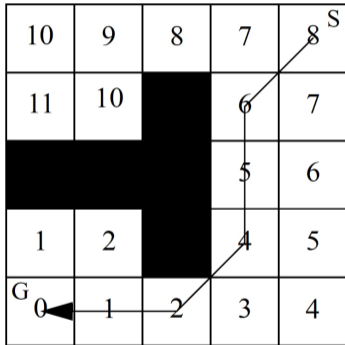
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- ▶ These algorithms are called **deterministic search algorithm**
- ▶ More details about them are discussed in the AI course

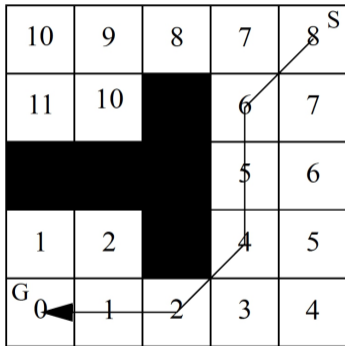


# Wavefront Algorithm



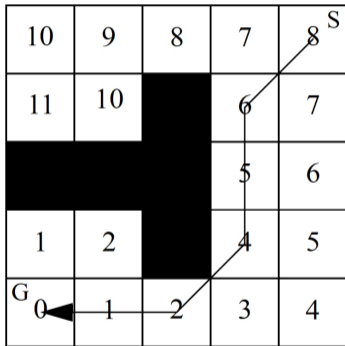
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- ▶ The algorithm **starts the search process from the goal** and **stops when a robot's initial position is reached**
- ▶ An important outcome of the wavefront algorithm is an **estimate of the distance from any expanded node to the goal** (represented as a Manhattan distance)

# Rapidly Exploring Random Trees (RRTs)



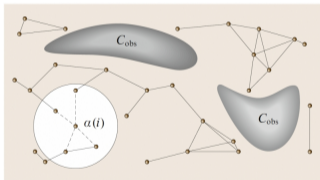
- ▶ Many robot planning tasks, particularly in high dimensions, are done using **randomised search**
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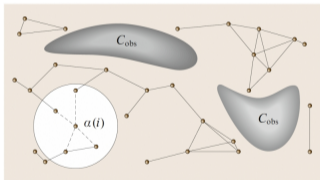


- ▶ RRT is one such algorithm that, at each step, **randomly select a free space node  $q'$**  and connects that to already an existing graph segment **if the connection leads to a collision-free path**
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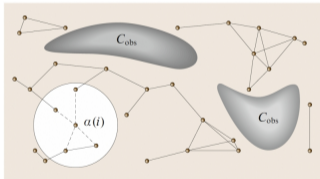


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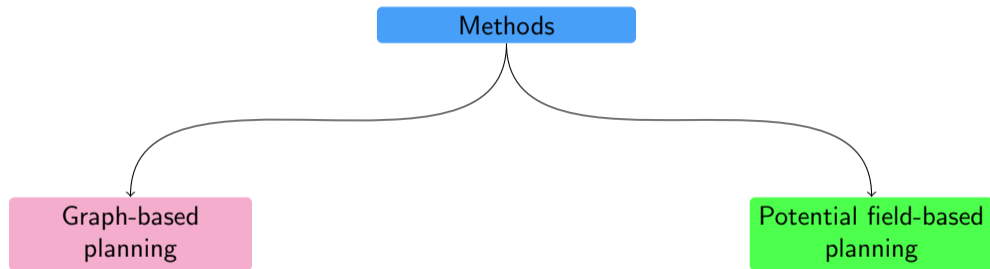
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- ▶ **RRT is a probabilistically complete algorithm** and **is not optimal**, but is fast and thus usually useful for practical purposes
  - ▶ The search typically needs to be repeated multiple times for a solution to be found

- ▶ As in some deterministic search algorithms, **the search process can be performed bidirectionally** (starting from both  $P_s$  and from  $P_g$ ) to increase the likelihood of finding a path

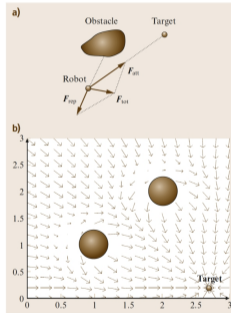
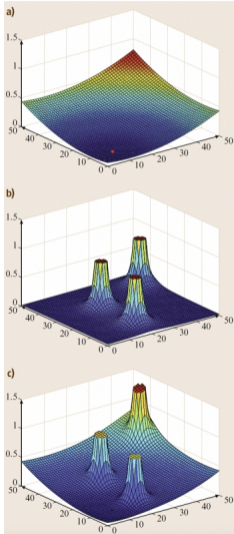
# Path Planning Methods



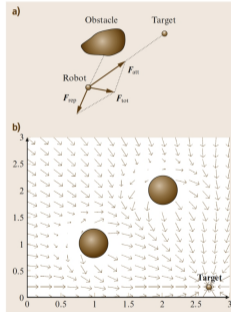
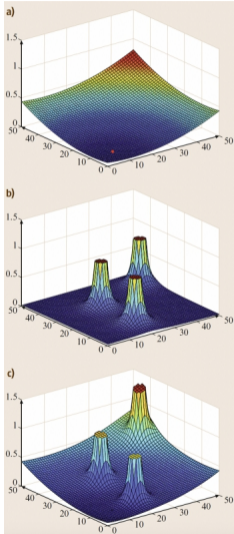
# Potential Fields



- ▶ Potential field planning is an alternative planning strategy based on which **the robot is treated as being under the influence of a potential field  $U(q)$**

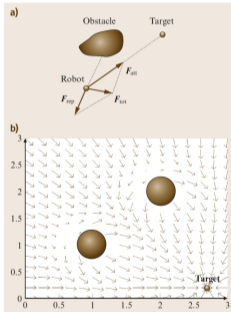
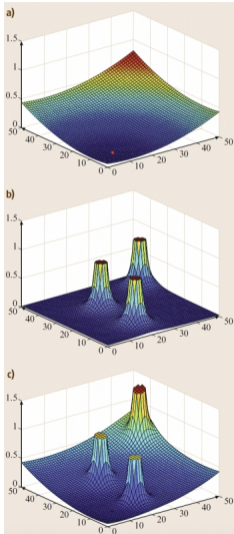


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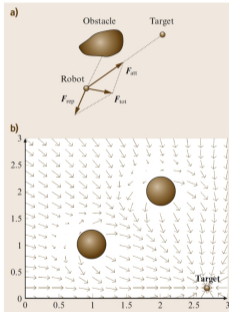
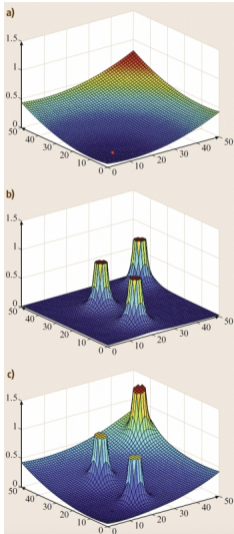
- ▶ Potential field planning is an alternative planning strategy based on which **the robot is treated as being under the influence of a potential field  $U(\mathbf{q})$**
- ▶  $U(\mathbf{q})$  is created as a combination of attractive and repulsive potentials:  $U(\mathbf{q}) = U_{attr}(\mathbf{q}) + U_{rep}(\mathbf{q})$ 
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$$\mathbf{F}(\mathbf{q}) = -\nabla U(\mathbf{q})$$
- ▶ This means that, at every point  $\mathbf{q}$ , a robot is subject to  $\mathbf{F}(\mathbf{q})$ , which dictates the direction in which the robot should move



- ▶ An attractive potential should **guide a robot towards a given configuration**
- ▶ Attractive potentials are typically used only for goal configurations; such a potential can be expressed **as a function of the distance to the goal**
- ▶ Let  $\|\mathbf{q} - \mathbf{q}_{goal}\|$  be the Euclidean distance between the current configuration and the goal configuration, and  $k_a$  be a positive constant; an example of an attractive potential would then be

$$U_{attr}(\mathbf{q}) = \frac{1}{2}k_a\|\mathbf{q} - \mathbf{q}_{goal}\|^2$$

- ▶ The associated force field is then

$$\mathbf{F}_{attr}(\mathbf{q}) = -\nabla U_{attr}(\mathbf{q}) = -k_a(\mathbf{q} - \mathbf{q}_{goal})$$

# Repulsive Potential

- ▶ A repulsive potential should **repel a robot from a given configuration**
- ▶ Repulsive potentials are typically used for avoiding obstacles, such that they can be expressed **as a function of the distance to obstacles** — each obstacle would have its own repulsive potential
- ▶ **Repulsive fields are typically active only within a given region** — faraway obstacles should not affect the motion of a robot
- ▶ Let  $\|\mathbf{q} - \mathbf{q}_o\|$  be the minimum distance between  $\mathbf{q}$  and any point of an obstacle,  $\rho_0$  be a distance threshold, and  $k_r$  a positive constant; an example of a repulsive field is then

$$U_{rep}(\mathbf{q}) = \begin{cases} \frac{1}{2}k_r \left( \frac{1}{\|\mathbf{q}-\mathbf{q}_o\|} - \frac{1}{\rho_0} \right)^2 & \|\mathbf{q} - \mathbf{q}_o\| \leq \rho_0 \\ 0 & \|\mathbf{q} - \mathbf{q}_o\| > \rho_0 \end{cases}$$

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# Potential Fields and Local Minima

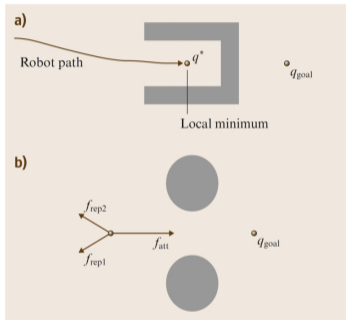


Fig.7.9a,b Two examples of the local minimum problem with potential functions

- ▶ Given the interplay between attractive and repulsive potentials, it can happen that the resulting force at a given point adds to **0** — **a robot gets stuck at a local minimum** in such a case

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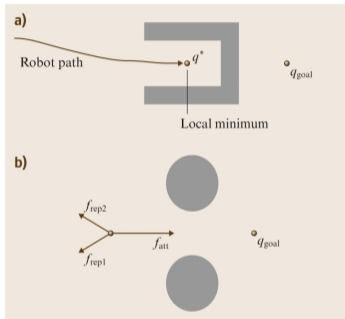


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- ▶ Given the interplay between attractive and repulsive potentials, it can happen that the resulting force at a given point adds to  $0$  — **a robot gets stuck at a local minimum** in such a case
- ▶ Thus, on their own, **a potential field is not a complete path planner**

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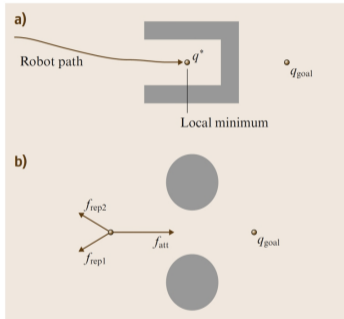
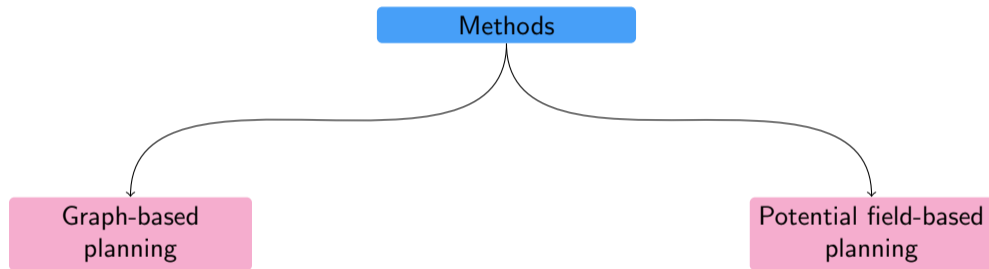


Fig.7.9a,b Two examples of the local minimum problem with potential functions

- ▶ Given the interplay between attractive and repulsive potentials, it can happen that the resulting force at a given point adds to **0** — **a robot gets stuck at a local minimum** in such a case
- ▶ Thus, on their own, **a potential field is not a complete path planner**
- ▶ One strategy to escape local minima is to employ **random walks** — this turns a potential field into a randomised planner

# Path Planning Methods



# Local Obstacle Avoidance



# Local Obstacle Avoidance for Unknown Obstacles



- ▶ Path planning can generate collision-free paths for known obstacles in the map, but **a robot should also have an ability to handle unknown and dynamic obstacles**
  - ▶ Very few environments are completely static — most are dynamic at least to some extent

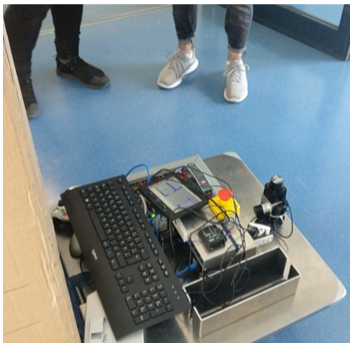


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- ▶ Local obstacle avoidance needs to **take the current sensor measurements into account** so that appropriate avoidance maneuvers can be performed

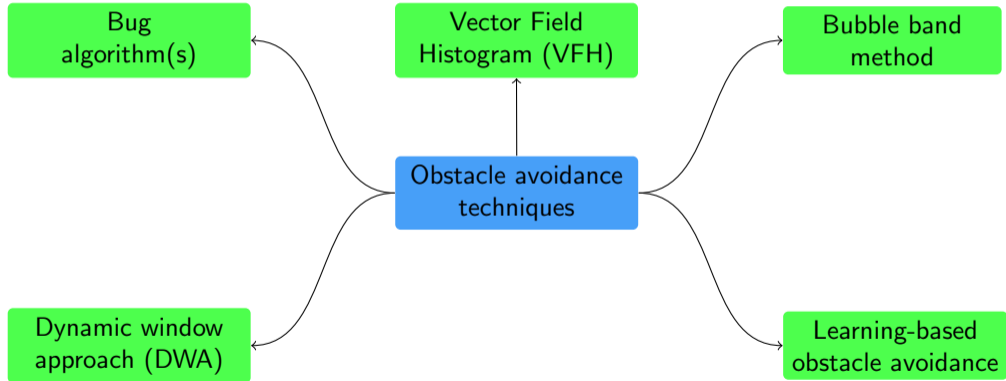
# Local Obstacle Avoidance for Unknown Obstacles



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  - ▶ Very few environments are completely static — most are dynamic at least to some extent
- ▶ Local obstacle avoidance needs to **take the current sensor measurements into account** so that appropriate avoidance maneuvers can be performed
- ▶ Traditional obstacle avoidance strategies are defined for **static obstacles** — dynamic obstacles (such as people) pose a different level of challenge and **are most effective in conjunction with an obstacle motion model**

# Obstacle Avoidance Techniques

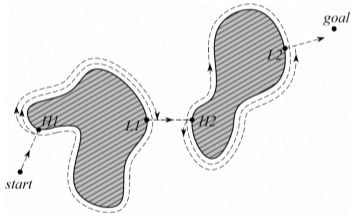
There is a large variety of obstacle avoidance techniques in the literature; we will take a closer look at some of them on the following slides



# Bug1 Algorithm



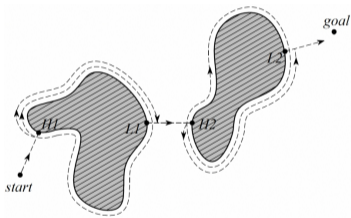
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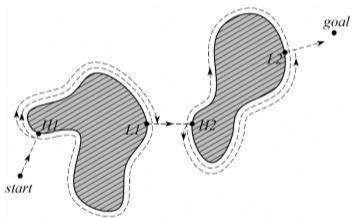
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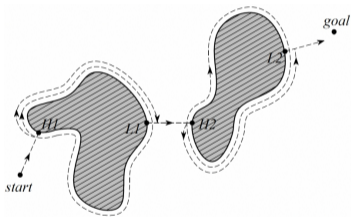
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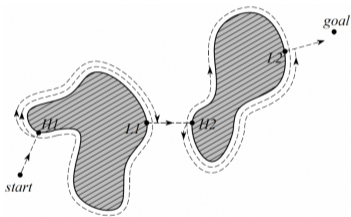
1. Circle around the obstacle (e.g. using wall following) to find the point that is closest to the goal and come back to the original obstacle approach point
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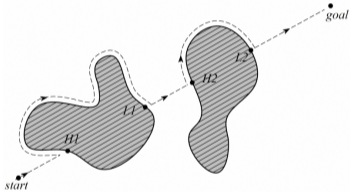


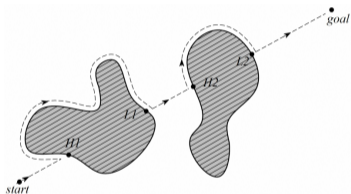
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  1. Circle around the obstacle (e.g. using wall following) to find the point that is closest to the goal and come back to the original obstacle approach point
  2. Follow the obstacle contour to the point closest to the goal and then leave the obstacle to move towards the goal
- ▶ Bug1 is a **naive and inefficient obstacle avoidance strategy**, as the full obstacle contour needs to be traversed so that a departure point is identified



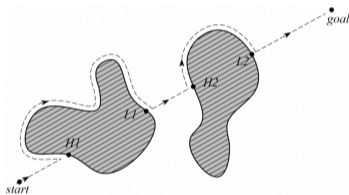


- ▶ Bug2 constitutes a more efficient version of Bug1



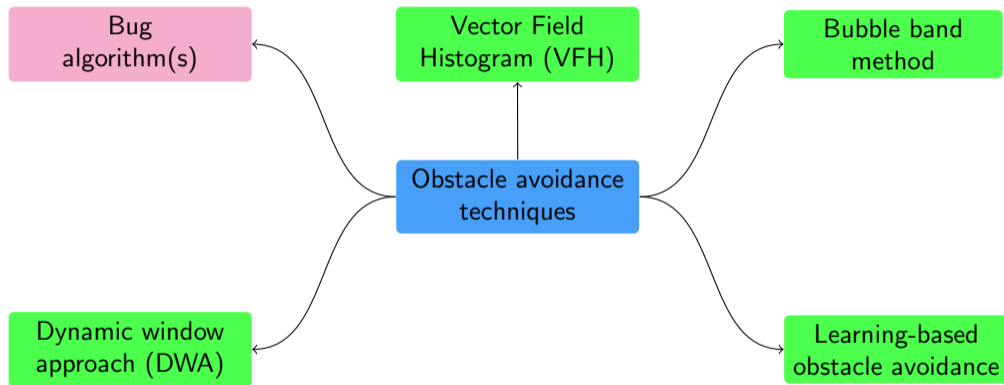


- ▶ Bug2 constitutes a more efficient version of Bug1
- ▶ The idea behind Bug2 is to **follow the obstacle's contour until reaching a point from which there is a direct path to the goal**; at this point, the robot leaves the obstacle and starts moving towards the goal



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- ▶ The idea behind Bug2 is to **follow the obstacle's contour until reaching a point from which there is a direct path to the goal**; at this point, the robot leaves the obstacle and starts moving towards the goal
- ▶ Some non-convex obstacle shapes may lead to a suboptimal or oscillatory behaviour of the bug algorithms

# Obstacle Avoidance Techniques



# Vector Field Histogram (VFH)

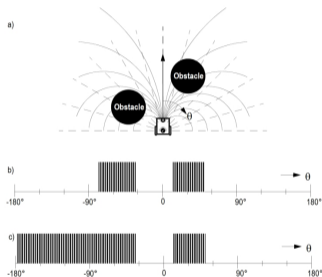
- ▶ A vector field histogram is an obstacle avoidance method that uses a local map based on recent sensor measurements



# Vector Field Histogram (VFH)

- ▶ A vector field histogram is an obstacle avoidance method that uses a local map based on recent sensor measurements

- ▶ The method **creates a discrete histogram that encodes the probability that there is an obstacle at a given direction from the robot**



**Figure 6.17**  
Example of blocked directions and resulting polar histograms [54]. (a) Robot and blocking obstacles.  
(b) Polar histogram. (c) Masked polar histogram.

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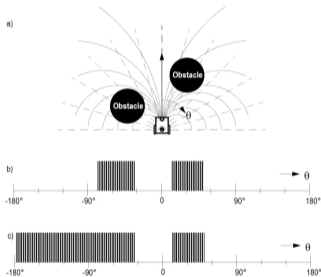


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Example of blocked directions and resulting polar histograms [54]. (a) Robot and blocking obstacles.  
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- ▶ The method **creates a discrete histogram that encodes the probability that there is an obstacle at a given direction from the robot**
- ▶ Given the histogram, **candidate passages that would fit the robot are found**, and a direction of motion is **identified based on a cost function** of the form:
$$J = w_1 h + w_2 \gamma + w_3 \Delta h$$
- ▶ Here  $w_{1,2,3}$  are positive constants,  $h$  is the orientation towards the goal,  $\gamma$  is the change in wheel orientation that would be necessary to move in the candidate orientation, and  $\Delta h$  is the necessary orientation change to achieve  $h$

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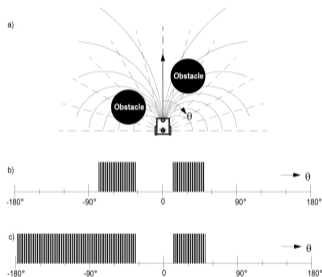


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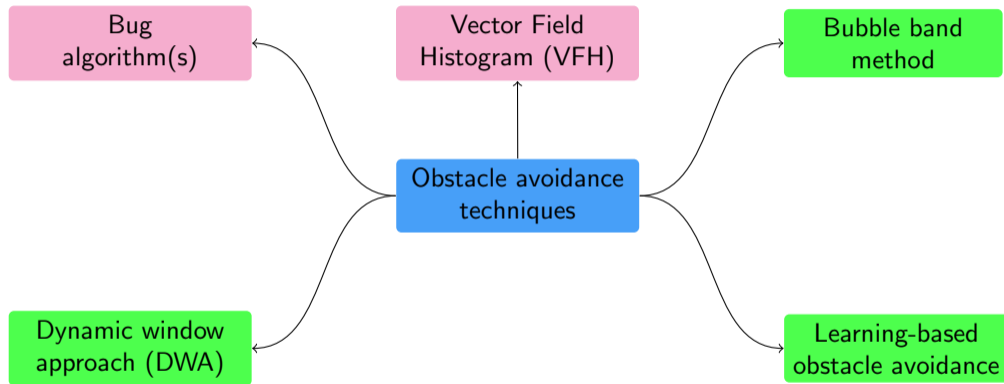
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- ▶ An extended VFH method assumes **motion along straight lines and arcs**, and creates a **masked histogram** that prevents motion directions that would pass through the obstacles



# Obstacle Avoidance Techniques



# Bubble Band Method

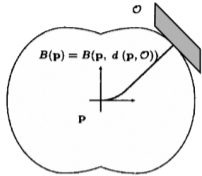


Figure 6.18  
Shape of the bubbles around the vehicle. Courtesy of Raja Chatila [165].

- ▶ The bubble band method **models the robot as a bubble**, where a bubble is **the maximum reachable space without collisions around a configuration  $q$**

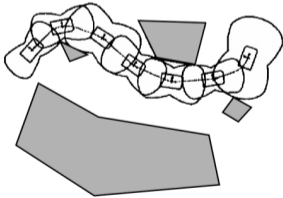


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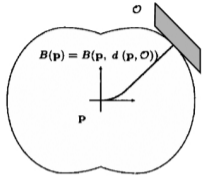


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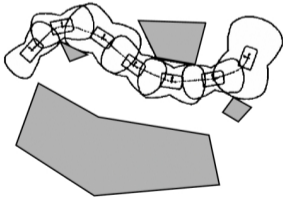


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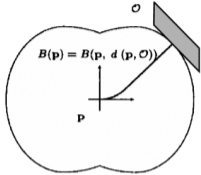


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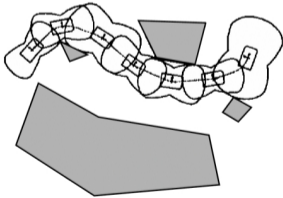


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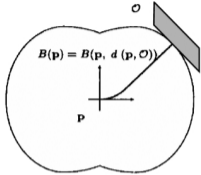


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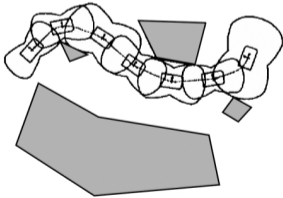
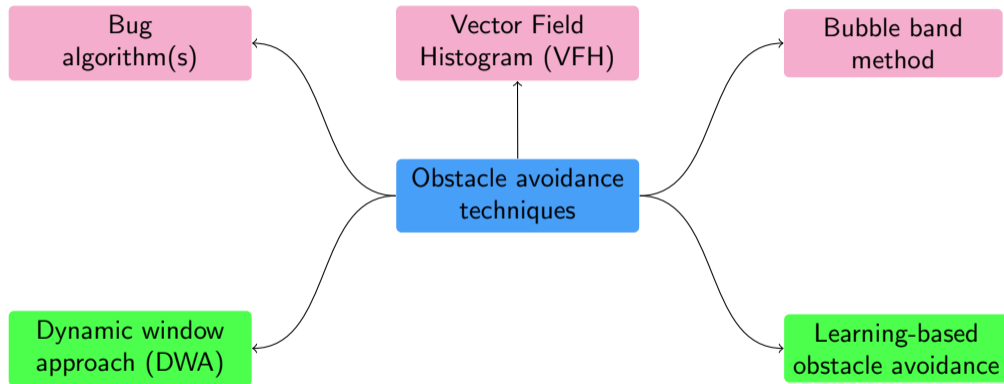


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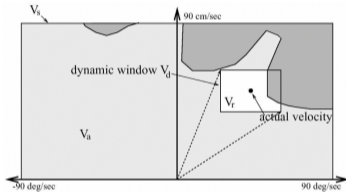
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- ▶ The bubble band method is thus **a path and motion planning method**

# Obstacle Avoidance Techniques



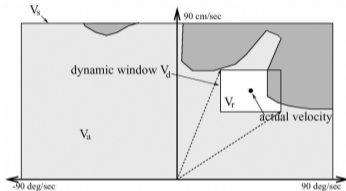
# Dynamic Window Approach (DWA)

- ▶ The dynamic window approach enables obstacle avoidance **by considering kinematic constraints**



**Figure 6.21**  
The dynamic window approach (courtesy of Dieter Fox [130]).  
The rectangular window shows the possible speeds ( $v$ ,  $\omega$ ) and the overlap with obstacles in configuration space.

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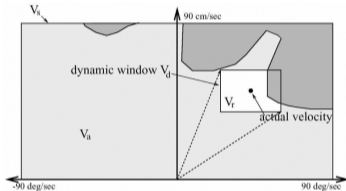


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- ▶ There are multiple variations of the technique, but they can roughly be divided into:
  - ▶ **Local DWA**, which only considers local obstacle information
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  - ▶ **Local DWA**, which only considers local obstacle information
  - ▶ **Global DWA**, which also includes global environment information in its planning process
- ▶ DWA is not just a method for path planning, but also for **motion planning**
  - ▶ Prediction of the effects of the robot's motion — based on a motion model — are thus done by the algorithm

# Local Dynamic Window Approach



- ▶ The local DWA assumes circular motion with linear velocity  $v$  and angular velocity  $\omega$ , such that it tries to **find instantaneous velocities that would bring the robot closer to the goal without causing an obstacle collision**



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- ▶ From the admissible set,  $v$  and  $\omega$  are chosen so that they **keep the robot as away from obstacles, are as aligned with the goal**, and **are as fast** as possible
  - ▶ This is achieved using an objective function of the form

$$J(v, \omega) = w_1 h(v, \omega) + w_2 s(v, \omega) + w_3 d(v, \omega)$$

where  $w_{1,2,3}$  are positive constants,  $h$  is the heading,  $s$  the speed, and  $d$  the closest distance to an obstacle

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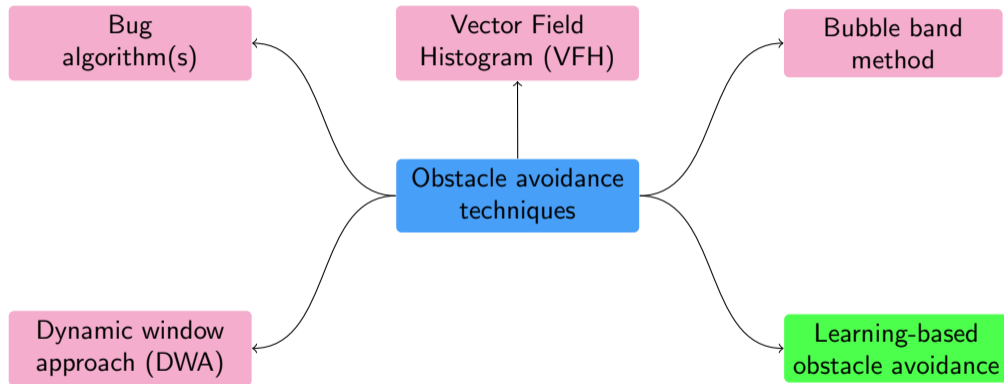


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- ▶ **The size of the region covered by the local occupancy grid is dynamically changed** so that the goal can always be found from the robot's current position
- ▶ The global DWA **reverts to the local DWA** when the robot is surrounded by obstacles and a path to the goal cannot be found using the wavefront algorithm

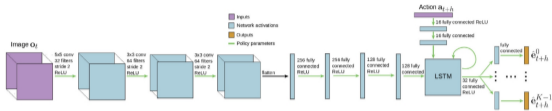




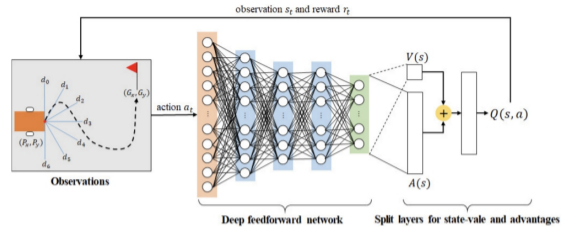
# Obstacle Avoidance Techniques



# Learning-Based Obstacle Avoidance



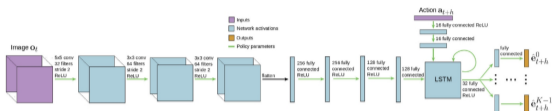
G. Kahn, P. Abbeel and S. Levine, "BADGR: An Autonomous Self-Supervised Learning-Based Navigation System," *IEEE Robotics and Automation Letters*, vol. 6, no. 2, pp. 1312–1319, Apr. 2021.



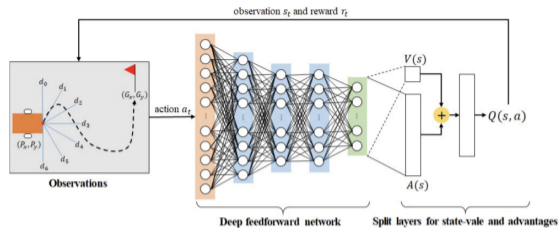
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# Learning-Based Obstacle Avoidance



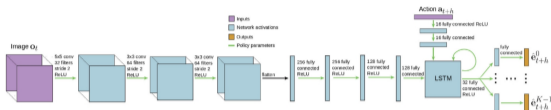
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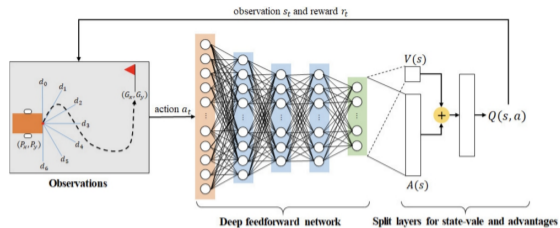
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# Learning-Based Obstacle Avoidance



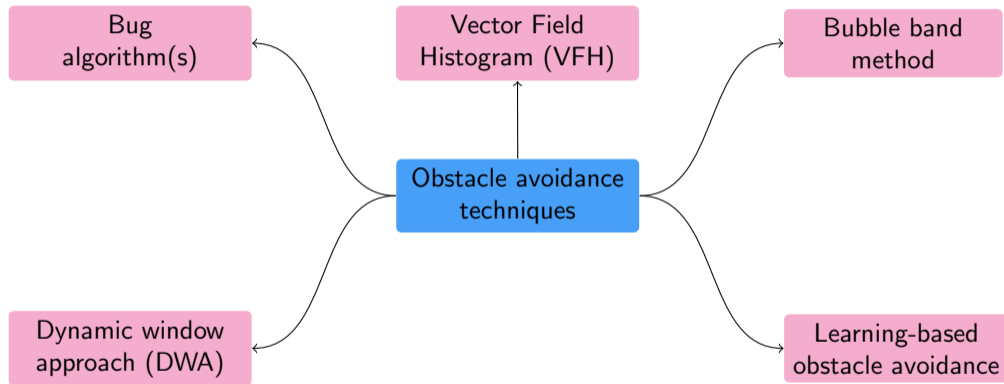
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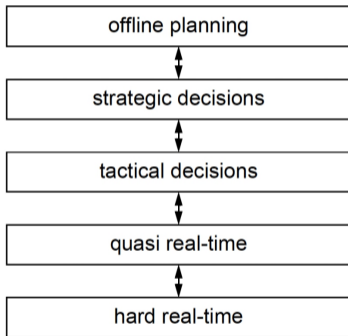
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- ▶ In recent years, there have been attempts to use learning algorithm that **acquire local navigation behaviours that map sensor measurements to motions** — often using **learned neural network-based policies**
- ▶ The development and exploration of such methods is, however, still an ongoing process — **model-based techniques still dominate navigation applications**

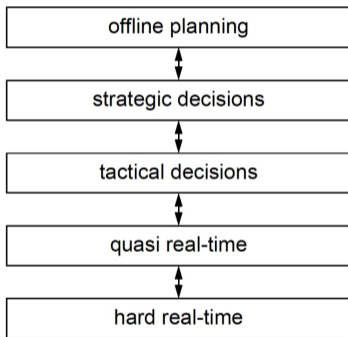
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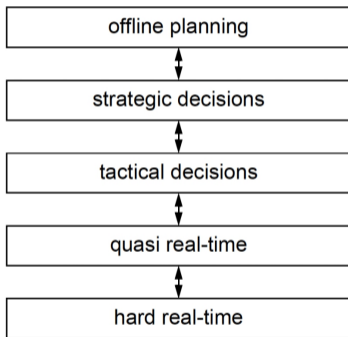
# Temporal Considerations



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  - ▶ operations where some latency (due to sensor processing or planning) can be tolerated (e.g. planning a path to a goal)
- ▶ Temporal constraints are taken into account within a **navigation architecture**



# Summary

- ▶ Path planning is the problem of finding a collision-free path that brings a robot from its initial location to a goal
- ▶ There are various (offline) path planning algorithms, which can be observed as belonging to two major categories: graph-based search and potential field planning
- ▶ Path planning algorithms find a path in a known map, but online obstacle avoidance is also required for dealing with environmental changes; there are many obstacle avoidance methods in the literature, most of which perform both path and motion planning (e.g. DWA)
- ▶ Machine learning-based approaches aim to replace the dependency on (simple) robot models by acquiring navigation behaviours from data
- ▶ Navigation architectures need to take into account timing constraints on the operation of a robot, particularly for functionalities that have hard real-time constraints