Hochschule
Bonn-Rhein-Sieg
University of Applied Sciences

b-it Bonn-Aachen International Center for Information Technology

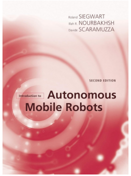
Institute for AI and Autonomous Systems

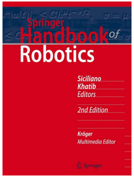
Kinematics

An Overview for Wheeled Robots

Dr. Alex Mitrevski
Master of Autonomous Systems

Structure





- Kinematics introduction
- Differential drive kinematics
- Motion constraints

Kinematics Introduction

What is Kinematics?

- For any mobile structure, it is important to **know how the structure can move around in space**
- The simplest way of studying motion is to **consider mathematical motion models that take into account the geometry of the structure and analyse motion under changing velocity** — this is the objective of kinematic models
- This is in contrast to **dynamics modelling, where forces / torques are considered as well**
- The primary use of kinematics models is that of **predicting the evolution of a robot's pose based on applied platform or individual wheel velocities**

Kinematics "relates to pure motion, i.e. to motion considered abstractly, without reference to force or mass" (Oxford Dictionary)

Forward vs. Inverse Kinematics Models

- When discussing kinematics models, it is important to note the difference between **forward and inverse kinematics models**

Forward vs. Inverse Kinematics Models

- When discussing kinematics models, it is important to note the difference between **forward and inverse kinematics models**

Forward kinematics

A forward kinematics model **predicts the motion of a mobile base as a result of the motion of the individual wheels**

NB: In the context of robot manipulation, forward kinematics allows calculating the pose of the end effector based on the velocities of the manipulator's joints

Forward vs. Inverse Kinematics Models

- When discussing kinematics models, it is important to note the difference between **forward and inverse kinematics models**

Forward kinematics

A forward kinematics model **predicts the motion of a mobile base as a result of the motion of the individual wheels**

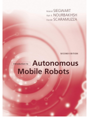
NB: In the context of robot manipulation, forward kinematics allows calculating the pose of the end effector based on the velocities of the manipulator's joints

Inverse kinematics

An inverse kinematics model **identifies the required motions of the individual wheels so that the mobile base can move in a particular way**

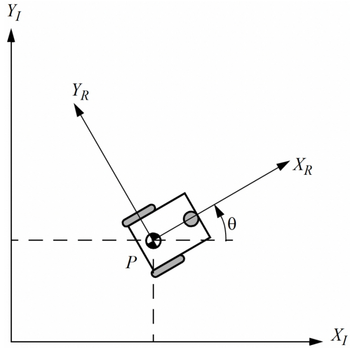
NB: In the manipulation context, inverse kinematics is concerned with calculating joint velocities that would result in a desired motion of the end effector

Robot Motion in Global and Local Coordinate Frames



- In a 2D plane, the pose ${}^{I}\boldsymbol{p}$ of a robot with respect to a global reference frame I is summarised by **the robot's position and planar orientation:** ${}^{I}\boldsymbol{p} = (x, y, \theta)^T$
- A robot's velocity vector ${}^{I}\dot{\boldsymbol{p}}$ with respect to I can be mapped to a velocity vector with respect to a robot's local frame R using a rotation matrix ${}^{R}_{I}R(\theta)$:

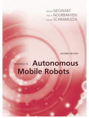
$$
{}^{R}\dot{\boldsymbol{p}} = {}^{R}_{I}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}
$$



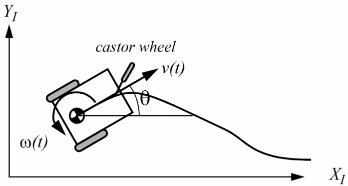
A robot with a local reference frame R at a reference point P, rotated about an angle θ with respect to a global reference frame I

Differential Drive Kinematics

Differential Drive Robot

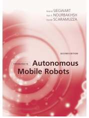


- A differential drive robot is a structure with **two active wheels**
  - Additional passive wheels may be included for stability
- The controllable wheels of a differential drive robot are **standard wheels** with an equal radius r
- We further consider that the wheels are **at a distance l from a point P that is centered between the wheels' axes**
- We consider **the velocities $\dot{\varphi}_1$ and $\dot{\varphi}_2$ of the wheels to be controllable**



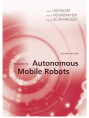
An illustration of a differential drive robot

Linear Motion



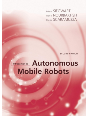
- First of all, note that, for a given angular velocity $\dot{\varphi}$, **a wheel with a radius r will have a forward linear velocity of $v_x = r\dot{\varphi}$**

Linear Motion



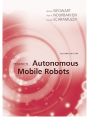
- First of all, note that, for a given angular velocity $\dot{\varphi}$, **a wheel with a radius r will have a forward linear velocity of $v_x = r\dot{\varphi}$**
- A differential drive has two wheels, both of which contribute to the motion:
  - If both wheels are moving with the same speed $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}$, the robot moves straight with a velocity $v_x = r\dot{\varphi}$

Linear Motion



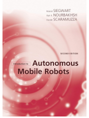
- First of all, note that, for a given angular velocity $\dot{\varphi}$, **a wheel with a radius r will have a forward linear velocity of $v_x = r\dot{\varphi}$**
- A differential drive has two wheels, both of which contribute to the motion:
  - If both wheels are moving with the same speed $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}$, the robot moves straight with a velocity $v_x = r\dot{\varphi}$
  - If one of the wheels is moving with a speed $\dot{\varphi}$ and the other one is stationary, the linear velocity of the robot will be $v_x = \frac{r\dot{\varphi}}{2}$

Linear Motion



- First of all, note that, for a given angular velocity $\dot{\varphi}$, **a wheel with a radius r will have a forward linear velocity of $v_x = r\dot{\varphi}$**
- A differential drive has two wheels, both of which contribute to the motion:
  - If both wheels are moving with the same speed $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}$, the robot moves straight with a velocity $v_x = r\dot{\varphi}$
  - If one of the wheels is moving with a speed $\dot{\varphi}$ and the other one is stationary, the linear velocity of the robot will be $v_x = \frac{r\dot{\varphi}}{2}$
  - If $\dot{\varphi}_2 = -\dot{\varphi}_1$, there is no linear motion — the robot rotates in place

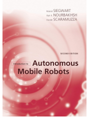
Linear Motion



- First of all, note that, for a given angular velocity $\dot{\varphi}$, **a wheel with a radius r will have a forward linear velocity of $v_x = r\dot{\varphi}$**
- A differential drive has two wheels, both of which contribute to the motion:
  - If both wheels are moving with the same speed $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}$, the robot moves straight with a velocity $v_x = r\dot{\varphi}$
  - If one of the wheels is moving with a speed $\dot{\varphi}$ and the other one is stationary, the linear velocity of the robot will be $v_x = \frac{r\dot{\varphi}}{2}$
  - If $\dot{\varphi}_2 = -\dot{\varphi}_1$, there is no linear motion — the robot rotates in place
- In general, **the contribution of the wheels is averaged to obtain the forward velocity**:

$$v_x = \frac{r\dot{\varphi}_1 + r\dot{\varphi}_2}{2}$$

Linear Motion



- First of all, note that, for a given angular velocity $\dot{\varphi}$, **a wheel with a radius r will have a forward linear velocity of $v_x = r\dot{\varphi}$**

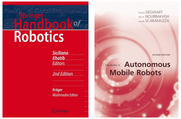
- A differential drive has two wheels, both of which contribute to the motion:
  - If both wheels are moving with the same speed $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}$, the robot moves straight with a velocity $v_x = r\dot{\varphi}$
  - If one of the wheels is moving with a speed $\dot{\varphi}$ and the other one is stationary, the linear velocity of the robot will be $v_x = \frac{r\dot{\varphi}}{2}$
  - If $\dot{\varphi}_2 = -\dot{\varphi}_1$, there is no linear motion — the robot rotates in place

- In general, **the contribution of the wheels is averaged to obtain the forward velocity**:

$$v_x = \frac{r\dot{\varphi}_1 + r\dot{\varphi}_2}{2}$$

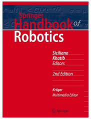
- A differential drive robot cannot move sideways, so $v_y = 0$

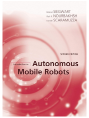
Angular Motion



- Robots rotate with a speed ω around a point called **instantaneous center of rotation (ICR)**
  - For a differential drive robot, if one of the wheels is static, the other wheel rotates in **a circular arc around the static wheel** (an arc of radius $2l$)

Angular Motion

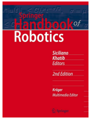


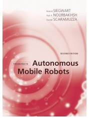


- Robots rotate with a speed ω around a point called **instantaneous center of rotation (ICR)**
  - For a differential drive robot, if one of the wheels is static, the other wheel rotates in **a circular arc around the static wheel** (an arc of radius $2l$)

- In general, **the angular motion depends on l** — the robot will rotate faster if l is shortened and slower if l is increased
  - This implies an inversely proportional relation between ω and l

Angular Motion

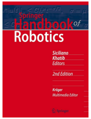


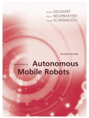


- Robots rotate with a speed ω around a point called **instantaneous center of rotation (ICR)**
  - For a differential drive robot, if one of the wheels is static, the other wheel rotates in **a circular arc around the static wheel** (an arc of radius $2l$)
- In general, **the angular motion depends on l** — the robot will rotate faster if l is shortened and slower if l is increased
  - This implies an inversely proportional relation between ω and l
- The overall ω combines the individual wheel contributions:

$$\omega = \frac{r\dot{\varphi}_1 - r\dot{\varphi}_2}{2l}$$

Angular Motion





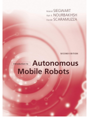
- Robots rotate with a speed ω around a point called **instantaneous center of rotation (ICR)**
  - For a differential drive robot, if one of the wheels is static, the other wheel rotates in **a circular arc around the static wheel** (an arc of radius $2l$)
- In general, **the angular motion depends on l** — the robot will rotate faster if l is shortened and slower if l is increased
  - This implies an inversely proportional relation between ω and l
- The overall ω combines the individual wheel contributions:

$$\omega = \frac{r\dot{\varphi}_1 - r\dot{\varphi}_2}{2l}$$

- The distance D of P to the ICR can also be found as

$$D = l\frac{r\dot{\varphi}_1 + r\dot{\varphi}_2}{r\dot{\varphi}_1 - r\dot{\varphi}_2}$$

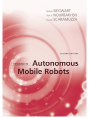
Forward Kinematics Model



- The complete forward kinematics model of a differential drive platform is given as

$$
{}^{R}\dot{\boldsymbol{p}} = \begin{pmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{pmatrix}
$$

Forward Kinematics Model



- The complete forward kinematics model of a differential drive platform is given as

$$ {}^{R}\dot{\boldsymbol{p}} = \begin{pmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{pmatrix} $$

- With respect to the global reference frame I, the model is given as

$$ {}^{I}\dot{\boldsymbol{p}} = R(\theta)^{T}\,{}^{R}\dot{\boldsymbol{p}} = R(\theta)^{T} \begin{pmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{pmatrix} $$

Motion Constraints

Motion Constraints

- On a mobile robot base, the overall motion of the robot is a result of the motions of the individual wheels
  - This implies that the motion of individual wheels of a mobile base **may constrain the motion of the robot's base** in certain ways

Motion Constraints

- On a mobile robot base, the overall motion of the robot is a result of the motions of the individual wheels
  - This implies that the motion of individual wheels of a mobile base **may constrain the motion of the robot's base** in certain ways
- Considering such constraints makes it possible to completely capture the motion of the base during kinematic modelling

Motion Constraints

- On a mobile robot base, the overall motion of the robot is a result of the motions of the individual wheels
  - This implies that the motion of individual wheels of a mobile base **may constrain the motion of the robot's base** in certain ways
- Considering such constraints makes it possible to completely capture the motion of the base during kinematic modelling
- There are two concrete **constraints on the instantaneous wheel motion** that can provide useful information about a robot's motion:

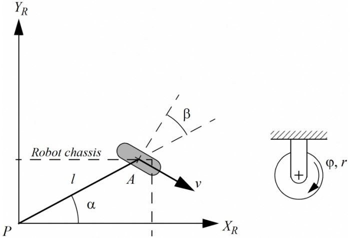
Motion Constraints

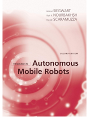
- On a mobile robot base, the overall motion of the robot is a result of the motions of the individual wheels
  - This implies that the motion of individual wheels of a mobile base **may constrain the motion of the robot's base** in certain ways
- Considering such constraints makes it possible to completely capture the motion of the base during kinematic modelling
- There are two concrete **constraints on the instantaneous wheel motion** that can provide useful information about a robot's motion:
  - **Rolling constraint**, which specifies a wheel's rolling motion (motion parallel to the wheel's plane) with respect to P

Motion Constraints

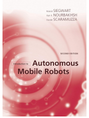
- On a mobile robot base, the overall motion of the robot is a result of the motions of the individual wheels
  - This implies that the motion of individual wheels of a mobile base **may constrain the motion of the robot's base** in certain ways
- Considering such constraints makes it possible to completely capture the motion of the base during kinematic modelling
- There are two concrete **constraints on the instantaneous wheel motion** that can provide useful information about a robot's motion:
  - **Rolling constraint**, which specifies a wheel's rolling motion (motion parallel to the wheel's plane) with respect to P
  - **Sliding constraint**, which specifies a wheel's sideways sliding motion (motion orthogonal to the wheel's plane) with respect to P

Standard Wheel Constraints





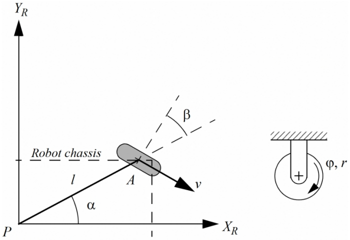
Standard Wheel Constraints



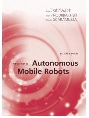
- For a fixed standard wheel, the motion parallel to the wheel's plane is captured by the wheel's rolling, so

Rolling constraint for a fixed standard wheel

$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^{R}_{I}R(\theta)^{I}\dot{\boldsymbol{p}} = r\dot{\varphi}$$



Standard Wheel Constraints



- For a fixed standard wheel, the motion parallel to the wheel's plane is captured by the wheel's rolling, so

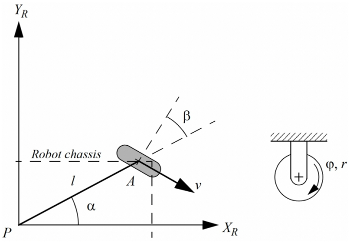
Rolling constraint for a fixed standard wheel

$$\left(\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos(\beta)\right) {}^{R}_{I}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = r\dot{\varphi}$$

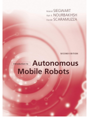
- A standard wheel cannot slide sideways; thus, the velocity orthogonal to the wheel's plane needs to be equal to 0:

Sliding constraint for a fixed standard wheel

$$\left(\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin(\beta)\right) {}^{R}_{I}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = 0$$



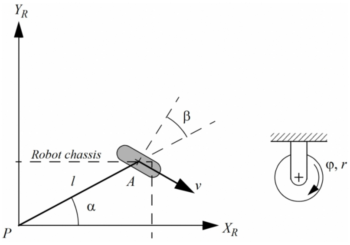
Standard Wheel Constraints



- For a fixed standard wheel, the motion parallel to the wheel's plane is captured by the wheel's rolling, so

Rolling constraint for a fixed standard wheel

$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^{R}_{I}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = r\dot{\varphi}$$



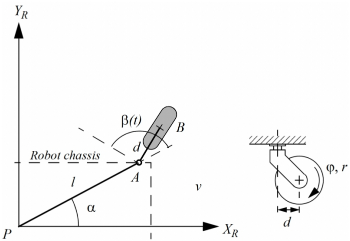
- A standard wheel cannot slide sideways; thus, the velocity orthogonal to the wheel's plane needs to be equal to 0:

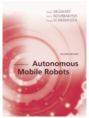
Sliding constraint for a fixed standard wheel

$$\begin{pmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin(\beta)\end{pmatrix} {}^{R}_{I}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = 0$$

- The sliding and rolling constraint are the same for a steered standard wheel (with a variable β)

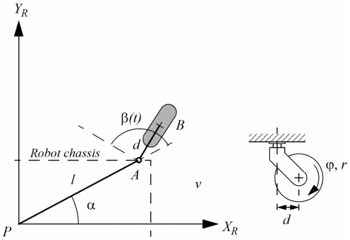
Caster Wheel Constraints





Caster Wheel Constraints

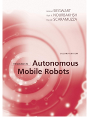
- A caster wheel differs from a steered standard wheel in its vertical rotation axis, which **does not coincide with the wheel's contact point with the ground**
- This offset has no effect on the rolling constraint on the wheel:



Rolling constraint for a caster wheel

$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

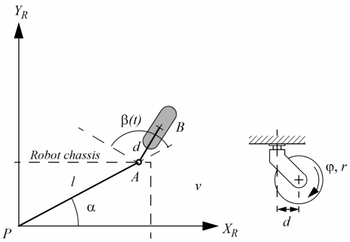
Caster Wheel Constraints



- A caster wheel differs from a steered standard wheel in its vertical rotation axis, which **does not coincide with the wheel's contact point with the ground**
- This offset has no effect on the rolling constraint on the wheel:

Rolling constraint for a caster wheel

$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

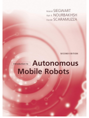


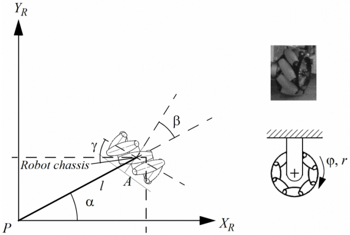
- The offset allows lateral motion, such that a steering motion can counteract any motion orthogonal to the wheel's axis (caused by a force at the caster attachment point A):

Sliding constraint for a caster wheel

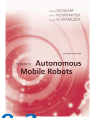
$$\begin{pmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & d+l\sin(\beta)\end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = -d\dot{\beta}$$

Swedish Wheel Constraints



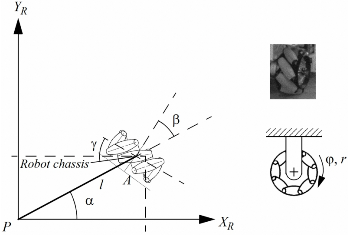


Swedish Wheel Constraints



- Unlike a standard wheel, **a Swedish wheel does not have a vertical axis of rotation**

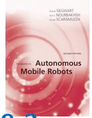
- Its rolling constraint is expressed with respect to an axis at the contact point of the rollers with the ground, which is where rolling occurs:



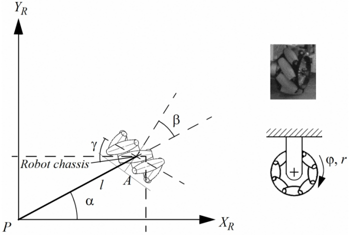
Rolling constraint for a Swedish wheel

$$\left(\sin(\alpha+\beta+\gamma) \quad -\cos(\alpha+\beta+\gamma) \quad -l\cos(\beta+\gamma)\right) {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}\cos(\gamma)$$

Swedish Wheel Constraints



- Unlike a standard wheel, **a Swedish wheel does not have a vertical axis of rotation**



- Its rolling constraint is expressed with respect to an axis at the contact point of the rollers with the ground, which is where rolling occurs:

Rolling constraint for a Swedish wheel

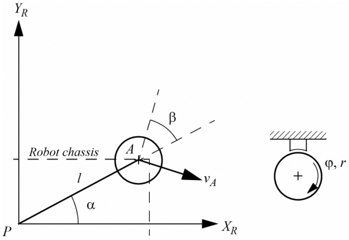
$$\begin{pmatrix}\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l\cos(\beta+\gamma)\end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}\cos(\gamma)$$

- As rollers can rotate freely with a velocity $\dot{\varphi}_{sw}$, the orthogonal motion with respect to the roller axis is not constrained; this is expressed as

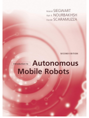
Sliding constraint for a Swedish wheel

$$\begin{pmatrix}\cos(\alpha+\beta+\gamma) & \sin(\alpha+\beta+\gamma) & l\sin(\beta+\gamma)\end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}\sin(\gamma) + r_{sw}\dot{\varphi}_{sw}$$

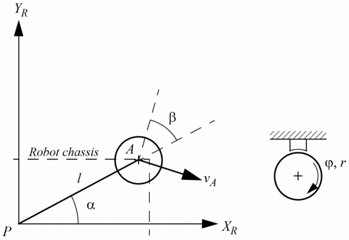
Spherical Wheel Constraints



Spherical Wheel Constraints



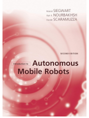
- A spherical wheel has no concrete axis of rotation, so its constraints are defined based on conventions
- The rolling constraint expresses the rolling in the wheel's direction of motion:

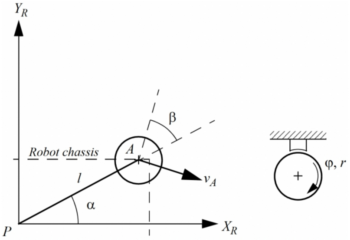


Rolling constraint for a spherical wheel

$$\begin{pmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta) \end{pmatrix} {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

Spherical Wheel Constraints





- A spherical wheel has no concrete axis of rotation, so its constraints are defined based on conventions
- The rolling constraint expresses the rolling in the wheel's direction of motion:

Rolling constraint for a spherical wheel

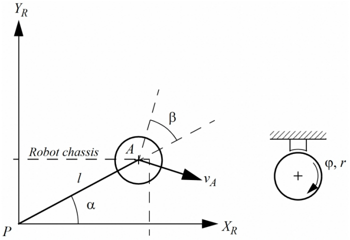
$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

- The sliding constraint expresses that the wheel does not move orthogonally to the direction of motion:

Sliding constraint for a spherical wheel

$$\begin{pmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin(\beta)\end{pmatrix} {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = 0$$

Spherical Wheel Constraints



- A spherical wheel has no concrete axis of rotation, so its constraints are defined based on conventions
- The rolling constraint expresses the rolling in the wheel's direction of motion:

Rolling constraint for a spherical wheel

$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^R_I R(\theta)^I \dot{\boldsymbol{p}} = r\dot{\varphi}$$

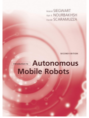
- The sliding constraint expresses that the wheel does not move orthogonally to the direction of motion:

Sliding constraint for a spherical wheel

$$\begin{pmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin(\beta)\end{pmatrix} {}^R_I R(\theta)^I \dot{\boldsymbol{p}} = 0$$

- NB: Mathematically, these are the same as for a standard wheel, but they are interpreted differently because **a spherical wheel does not pose any constraints on a robot's motion**

Kinematic Constraints on a Mobile Base



- To express the kinematic constraints of a mobile robot base, **only fixed and steered standard wheels need to be taken into account** — caster, Swedish, and spherical wheels put no constraints on the motion

- Let us consider **a mobile robot base with $N = N_f + N_s$ standard wheels**, with N_f the number of fixed standard wheels and N_s the number of steered standard wheels
  - We combine the motion of these into a common vector $\dot{\varphi} = (\dot{\varphi}_f \ \dot{\varphi}_s)^T$

- The rolling and sliding constraints for the base are expressed by combining the constraints of the standard wheels

Mobile Base Rolling Constraints

- The rolling constraint of a mobile base is **a system that combines the rolling constraints of the N wheels**, which is expressed as

$$J_1(\beta_s) {}^R_I R(\theta) {}^I\boldsymbol{p} = J_2 \dot{\varphi}$$

- The matrices J_1 and J_2 are given as

$$J_1(\beta_s) = \begin{pmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{pmatrix} \qquad J_2 = \begin{pmatrix} r_{f_1} & & & & & \\ & \ddots & & & & \\ & & r_{f_{N_f}} & & & \\ & & & r_{s_1} & & \\ & & & & \ddots & \\ & & & & & r_{s_{N_s}} \end{pmatrix}$$

Mobile Base Sliding Constraints

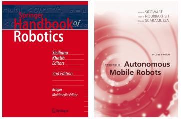
- The sliding constraint is also **a system combining the sliding constraints of the N standard wheels**; this is expressed as follows:

$$C(\beta_s)\,{}^R_I R(\theta)\,{}^I\boldsymbol{p} = \mathbf{0}$$

where

$$C(\beta_s) = \begin{pmatrix} C_f \\ C_s(\beta_s) \end{pmatrix}$$

Instantaneous Center of Rotation (ICR)



- The point around which a mobile robot base rotates is determined by the constraints imposed by its wheels
- This point is the **instantaneous center of rotation (ICR)** that we already encountered for a differential drive robot
- The ICR can be determined by drawing motion lines perpendicularly through the wheel axes; **the intersection point of these lines is the ICR**
  - If the wheels move in a straight line, the ICR is at infinity
- It should be noted that **not all standard wheels provide independent constraints for determining the ICR**
  - For a car, the back wheels are fixed along the same axis, so they do not both independently constrain the vehicle's motion

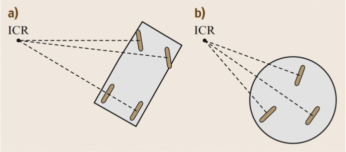
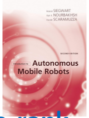


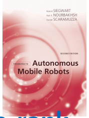
Fig.24.5a,b The instantaneous center of rotation. **(a)** A car-like robot; **(b)** a three-steering-wheels robot

Degree of Mobility



- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\mathrm{rank}(C(\beta_s))$
  - As a wheeled robot moves on a plane, we have that $0 \leq \mathrm{rank}(C(\beta_s)) \leq 3$

Degree of Mobility

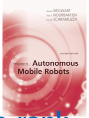


- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\mathrm{rank}(C(\beta_s))$
  - As a wheeled robot moves on a plane, we have that $0 \leq \mathrm{rank}(C(\beta_s)) \leq 3$

- The **degree of mobility** δ_m of a mobile robot is defined in terms of the null space of $C(\beta_s)$, which is a function of $\mathrm{rank}(C(\beta_s))$:

$$\delta_m = \dim\left(\mathrm{null}\left(C(\beta_s)\right)\right) = 3 - \mathrm{rank}\left(C(\beta_s)\right)$$

Degree of Mobility



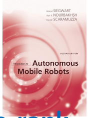
- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\text{rank}(C(\beta_s))$
  - As a wheeled robot moves on a plane, we have that $0 \leq \text{rank}(C(\beta_s)) \leq 3$

- The **degree of mobility** δ_m of a mobile robot is defined in terms of the null space of $C(\beta_s)$, which is a function of $\text{rank}(C(\beta_s))$:

$$\delta_m = \dim\left(\text{null}\left(C(\beta_s)\right)\right) = 3 - \text{rank}\left(C(\beta_s)\right)$$

- We can consider the following examples resulting from this definition:
  - A bicycle has $\delta_m = 1$: at any point, it can either move forward or change its rotation

Degree of Mobility



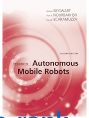
- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\mathrm{rank}(C(\beta_s))$
  - As a wheeled robot moves on a plane, we have that $0 \leq \mathrm{rank}(C(\beta_s)) \leq 3$

- The **degree of mobility** δ_m of a mobile robot is defined in terms of the null space of $C(\beta_s)$, which is a function of $\mathrm{rank}(C(\beta_s))$:

$$\delta_m = \dim\left(\mathrm{null}\left(C(\beta_s)\right)\right) = 3 - \mathrm{rank}\left(C(\beta_s)\right)$$

- We can consider the following examples resulting from this definition:
  - A bicycle has $\delta_m = 1$: at any point, it can either move forward or change its rotation
  - A differential drive has $\delta_m = 2$: both the linear speed and orientation can be controlled instantaneously

Degree of Mobility



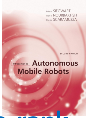
- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\text{rank}(C(\beta_s))$
  - As a wheeled robot moves on a plane, we have that $0 \leq \text{rank}(C(\beta_s)) \leq 3$

- The **degree of mobility** δ_m of a mobile robot is defined in terms of the null space of $C(\beta_s)$, which is a function of $\text{rank}(C(\beta_s))$:

$$\delta_m = \dim\left(\text{null}\left(C(\beta_s)\right)\right) = 3 - \text{rank}\left(C(\beta_s)\right)$$

- We can consider the following examples resulting from this definition:
  - A bicycle has $\delta_m = 1$: at any point, it can either move forward or change its rotation
  - A differential drive has $\delta_m = 2$: both the linear speed and orientation can be controlled instantaneously
  - A robot without standard wheels has $\delta_m = 3$

Degree of Mobility

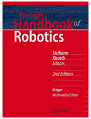


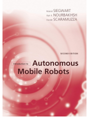
- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\mathrm{rank}(C(\beta_s))$
  - As a wheeled robot moves on a plane, we have that $0 \leq \mathrm{rank}(C(\beta_s)) \leq 3$
- The **degree of mobility** δ_m of a mobile robot is defined in terms of the null space of $C(\beta_s)$, which is a function of $\mathrm{rank}(C(\beta_s))$:

$$\delta_m = \dim\left(\mathrm{null}\left(C(\beta_s)\right)\right) = 3 - \mathrm{rank}\left(C(\beta_s)\right)$$

- We can consider the following examples resulting from this definition:
  - A bicycle has $\delta_m = 1$: at any point, it can either move forward or change its rotation
  - A differential drive has $\delta_m = 2$: both the linear speed and orientation can be controlled instantaneously
  - A robot without standard wheels has $\delta_m = 3$
- The degree of mobility determines the **differentiable degrees of freedom** (DDOF), which is the number of velocities that can be independently achieved by a robot

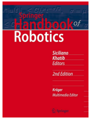
Degree of Steerability

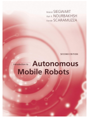




- The **degree of steerability** δ_s of a mobile robot is defined as the number of steerable parameters that can be independently controlled

Degree of Steerability

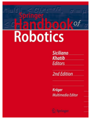


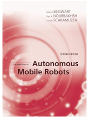


- The **degree of steerability** δ_s of a mobile robot is defined as the number of steerable parameters that can be independently controlled
- This also depends on the number of independent constraints imposed by the wheels on the base and is **defined via the rank of** $C_s(\beta_s)$:

$$\delta_s = \text{rank}\left(C_s(\beta_s)\right)$$

Degree of Steerability



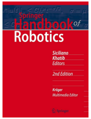


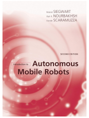
- The **degree of steerability** δ_s of a mobile robot is defined as the number of steerable parameters that can be independently controlled
- This also depends on the number of independent constraints imposed by the wheels on the base and is **defined via the rank of** $C_s(\beta_s)$:

$$\delta_s = \text{rank}\left(C_s(\beta_s)\right)$$

- Steerable wheels decrease the mobility (they impose a sliding constraint), but enable a robot to adapt its overall motion through the steerable orientation

Degree of Steerability



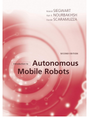


- The **degree of steerability** δ_s of a mobile robot is defined as the number of steerable parameters that can be independently controlled
- This also depends on the number of independent constraints imposed by the wheels on the base and is **defined via the rank of** $C_s(\beta_s)$:

$$\delta_s = \text{rank}\left(C_s(\beta_s)\right)$$

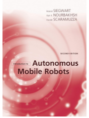
- Steerable wheels decrease the mobility (they impose a sliding constraint), but enable a robot to adapt its overall motion through the steerable orientation
- δ_s is typically defined so that $0 \leq \delta_s \leq 2$; if there are more than two steered wheels, the existence of an ICR may not be guaranteed!

Degree of Maneuverability



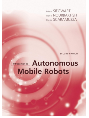
- The **degree of maneuverability** δ_M defines the total number of degrees of freedom that can be controlled by a wheeled mobile robot

Degree of Maneuverability



- The **degree of maneuverability** δ_M defines the total number of degrees of freedom that can be controlled by a wheeled mobile robot
- Intuitively, δ_M summarises:
  - the degrees of freedom that can be controlled directly through the individual wheel velocities and
  - the degrees of freedom whose control is made possible through steering

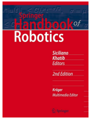
Degree of Maneuverability

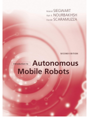


- The **degree of maneuverability** δ_M defines the total number of degrees of freedom that can be controlled by a wheeled mobile robot
- Intuitively, δ_M summarises:
  - the degrees of freedom that can be controlled directly through the individual wheel velocities and
  - the degrees of freedom whose control is made possible through steering
- δ_M is thus defined as a combination of the degrees of mobility and steerability:

$$\delta_M = \delta_m + \delta_s$$

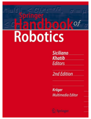
Basic Types of Wheeled Mobile Robots

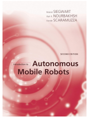




Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels

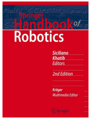
Basic Types of Wheeled Mobile Robots

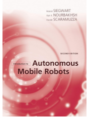




Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels
(2,0)	$\delta_M = 2,\ \delta_m = 2,\ \delta_s = 0$	One or more fixed wheels with a common axis; no steering wheels

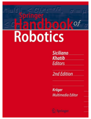
Basic Types of Wheeled Mobile Robots

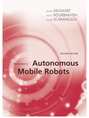




Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels
(2,0)	$\delta_M = 2,\ \delta_m = 2,\ \delta_s = 0$	One or more fixed wheels with a common axis; no steering wheels
(2,1)	$\delta_M = 3,\ \delta_m = 2,\ \delta_s = 1$	One or more steering wheels (if more than one, $\delta_s = 1$ needs to be ensured); no fixed wheels

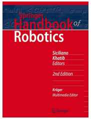
Basic Types of Wheeled Mobile Robots

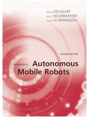




Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels
(2,0)	$\delta_M = 2,\ \delta_m = 2,\ \delta_s = 0$	One or more fixed wheels with a common axis; no steering wheels
(2,1)	$\delta_M = 3,\ \delta_m = 2,\ \delta_s = 1$	One or more steering wheels (if more than one, $\delta_s = 1$ needs to be ensured); no fixed wheels
(1,1)	$\delta_M = 2,\ \delta_m = 1,\ \delta_s = 1$	One or more fixed wheels along a common axis; one or more steering wheels, not along the fixed axis and with coordinated motions

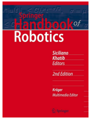
Basic Types of Wheeled Mobile Robots

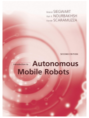




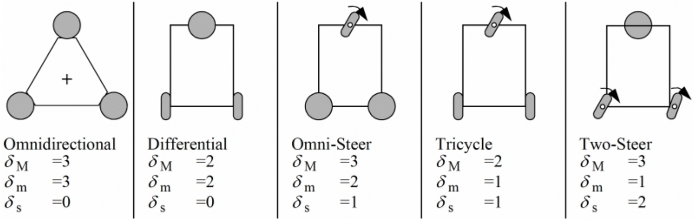
Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels
(2,0)	$\delta_M = 2,\ \delta_m = 2,\ \delta_s = 0$	One or more fixed wheels with a common axis; no steering wheels
(2,1)	$\delta_M = 3,\ \delta_m = 2,\ \delta_s = 1$	One or more steering wheels (if more than one, $\delta_s = 1$ needs to be ensured); no fixed wheels
(1,1)	$\delta_M = 2,\ \delta_m = 1,\ \delta_s = 1$	One or more fixed wheels along a common axis; one or more steering wheels, not along the fixed axis and with coordinated motions
(1,2)	$\delta_M = 3,\ \delta_m = 1,\ \delta_s = 2$	Two or more steering wheels; no fixed wheels

Basic Types of Wheeled Mobile Robots





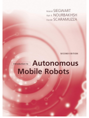
Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels
(2,0)	$\delta_M = 2,\ \delta_m = 2,\ \delta_s = 0$	One or more fixed wheels with a common axis; no steering wheels
(2,1)	$\delta_M = 3,\ \delta_m = 2,\ \delta_s = 1$	One or more steering wheels (if more than one, $\delta_s = 1$ needs to be ensured); no fixed wheels
(1,1)	$\delta_M = 2,\ \delta_m = 1,\ \delta_s = 1$	One or more fixed wheels along a common axis; one or more steering wheels, not along the fixed axis and with coordinated motions
(1,2)	$\delta_M = 3,\ \delta_m = 1,\ \delta_s = 2$	Two or more steering wheels; no fixed wheels



Holonomic and Nonholonomic Constraints

- In the context of constraints, we can make a distinction between holonomic and nonholonomic constraints

Holonomic and Nonholonomic Constraints

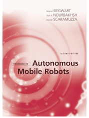


- In the context of constraints, we can make a distinction between holonomic and nonholonomic constraints

Holonomic constraint

A holonomic constraint is one that can be **expressed by only considering the position of a body, and potentially time as well**

Holonomic and Nonholonomic Constraints



- In the context of constraints, we can make a distinction between holonomic and nonholonomic constraints

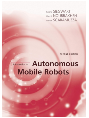
Holonomic constraint

A holonomic constraint is one that can be **expressed by only considering the position of a body, and potentially time as well**

Nonholonomic constraint

A nonholonomic constraint **cannot be expressed only through position variables**; instead, it is expressed as a differential constraint

Holonomic and Nonholonomic Constraints



- In the context of constraints, we can make a distinction between holonomic and nonholonomic constraints

Holonomic constraint

A holonomic constraint is one that can be **expressed by only considering the position of a body, and potentially time as well**

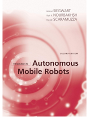
Nonholonomic constraint

A nonholonomic constraint **cannot be expressed only through position variables**; instead, it is expressed as a differential constraint

- Nonholonomic constraints are nonintegrable and thus express a dependence on the body's state on the path taken to reach that state

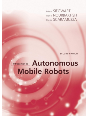
- The sliding constraint of standard wheels is a nonholonomic constraint

Workspace, Paths, Trajectories



When considering robots in terms of their control, it is important to consider what states they can (and cannot) achieve, and how

Workspace, Paths, Trajectories



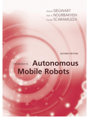
When considering robots in terms of their control, it is important to consider what states they can (and cannot) achieve, and how

Workspace

The workspace is the **set of all achievable configurations (poses) by a mobile robot**

- The dimensionality of the workspace is referred to as **the number of degrees of freedom (DOF)**

Workspace, Paths, Trajectories



When considering robots in terms of their control, it is important to consider what states they can (and cannot) achieve, and how

Workspace

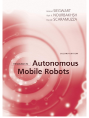
The workspace is the **set of all achievable configurations (poses) by a mobile robot**

- The dimensionality of the workspace is referred to as **the number of degrees of freedom (DOF)**

Path

A path is a (possibly infinite) **collection of poses that combine a starting pose with an end pose**

Workspace, Paths, Trajectories



When considering robots in terms of their control, it is important to consider what states they can (and cannot) achieve, and how

Workspace

The workspace is the **set of all achievable configurations (poses) by a mobile robot**

- The dimensionality of the workspace is referred to as **the number of degrees of freedom (DOF)**

Path

A path is a (possibly infinite) **collection of poses that combine a starting pose with an end pose**

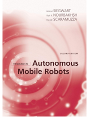
Trajectory

A trajectory combines a path with time, namely it is **a collection of poses along with time instants at which those poses should be reached**

What Does This All Mean in Practice?

- The maneuverability does tell us some important information about a mobile platform in terms of how it can be controlled, **but is not the main factor that determines whether a platform is practically useful**

What Does This All Mean in Practice?

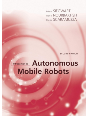


- The maneuverability does tell us some important information about a mobile platform in terms of how it can be controlled, **but is not the main factor that determines whether a platform is practically useful**
- In practice, additional factors need to be taken into account as well, such as **stability**, **maximum load that a platform can carry**, or **production cost**

What Does This All Mean in Practice?

- The maneuverability does tell us some important information about a mobile platform in terms of how it can be controlled, **but is not the main factor that determines whether a platform is practically useful**
- In practice, additional factors need to be taken into account as well, such as **stability**, **maximum load that a platform can carry**, or **production cost**
- Furthermore, various nonholonomic platforms (with $\delta_M < 3$) are able to achieve $DOF = 3$, so they are still practically useful
  - For instance, a car has $\delta_M = 2$, but it is a very useful mobile base (obviously)

What Does This All Mean in Practice?



- The maneuverability does tell us some important information about a mobile platform in terms of how it can be controlled, **but is not the main factor that determines whether a platform is practically useful**
- In practice, additional factors need to be taken into account as well, such as **stability**, **maximum load that a platform can carry**, or **production cost**
- Furthermore, various nonholonomic platforms (with $\delta_M < 3$) are able to achieve $DOF = 3$, so they are still practically useful
  - For instance, a car has $\delta_M = 2$, but it is a very useful mobile base (obviously)
- Practically speaking, **mobile bases with varying maneuverabilities differ in the manner in which they are controlled**: a base with lower maneuverability typically requires multiple maneuvers to follow certain paths and may not be able to follow arbitrary trajectories