Hochschule
Bonn-Rhein-Sieg
University of Applied Sciences

b-it Bonn-Aachen International Center for Information Technology

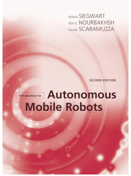
Institute for AI and Autonomous Systems

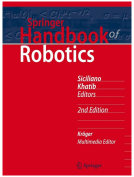
Kinematics

An Overview for Wheeled Robots

Dr. Alex Mitrevski
Master of Autonomous Systems

Structure





- Kinematics introduction
- Differential drive kinematics
- Motion constraints

Kinematics Introduction

What is Kinematics?

- For any mobile structure, it is important to **know how the structure can move around in space**
- The simplest way of studying motion is to **consider mathematical motion models that take into account the geometry of the structure and analyse motion under changing velocity** — this is the objective of kinematic models
- This is in contrast to **dynamics modelling, where forces / torques are considered as well**
- The primary use of kinematics models is that of **predicting the evolution of a robot's pose based on applied platform or individual wheel velocities**

Kinematics "relates to pure motion, i.e. to motion considered abstractly, without reference to force or mass" (Oxford Dictionary)

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Forward kinematics

A forward kinematics model **predicts the motion of a mobile base as a result of the motion of the individual wheels**

NB: In the context of robot manipulation, forward kinematics allows calculating the pose of the end effector based on the velocities of the manipulator's joints

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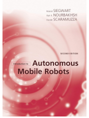
NB: In the context of robot manipulation, forward kinematics allows calculating the pose of the end effector based on the velocities of the manipulator's joints

Inverse kinematics

An inverse kinematics model **identifies the required motions of the individual wheels so that the mobile base can move in a particular way**

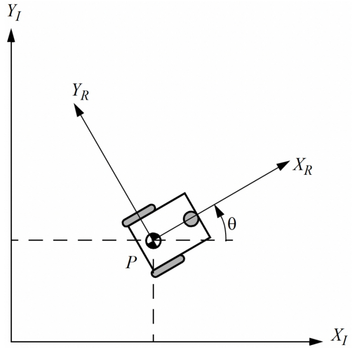
NB: In the manipulation context, inverse kinematics is concerned with calculating joint velocities that would result in a desired motion of the end effector

Robot Motion in Global and Local Coordinate Frames



- In a 2D plane, the pose ${}^{I}\boldsymbol{p}$ of a robot with respect to a global reference frame I is summarised by **the robot's position and planar orientation:** ${}^{I}\boldsymbol{p} = (x, y, \theta)^{T}$
- A robot's velocity vector ${}^{I}\dot{\boldsymbol{p}}$ with respect to I can be mapped to a velocity vector with respect to a robot's local frame R using a rotation matrix ${}_{I}^{R}R(\theta)$:

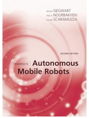
$$
{}^{R}\dot{\boldsymbol{p}} = {}_{I}^{R}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}
$$



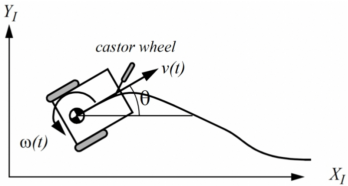
A robot with a local reference frame R at a reference point P, rotated about an angle θ with respect to a global reference frame I

Differential Drive Kinematics

Differential Drive Robot

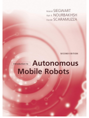


- A differential drive robot is a structure with **two active wheels**
  - Additional passive wheels may be included for stability
- The controllable wheels of a differential drive robot are **standard wheels** with an equal radius r
- We further consider that the wheels are **at a distance l from a point P that is centered between the wheels' axes**
- We consider **the velocities $\dot{\varphi}_1$ and $\dot{\varphi}_2$ of the wheels to be controllable**



An illustration of a differential drive robot

Linear Motion



- First of all, note that, for a given angular velocity $\dot{\varphi}$, **a wheel with a radius r will have a forward linear velocity of $v_x = r\dot{\varphi}$**

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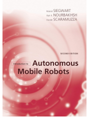

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- A differential drive has two wheels, both of which contribute to the motion:
  - If both wheels are moving with the same speed $\dot{\varphi}_1 = \dot{\varphi}_2 = \dot{\varphi}$, the robot moves straight with a velocity $v_x = r\dot{\varphi}$

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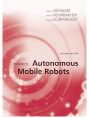

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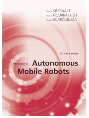

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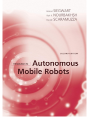

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$$v_x = \frac{r\dot{\varphi}_1 + r\dot{\varphi}_2}{2}$$

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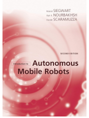

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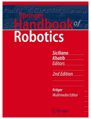
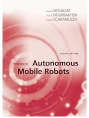
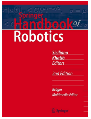
- A differential drive robot cannot move sideways, so $v_y = 0$

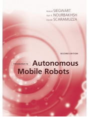




- Robots rotate with a speed ω around a point called **instantaneous center of rotation (ICR)**
  - For a differential drive robot, if one of the wheels is static, the other wheel rotates in **a circular arc around the static wheel** (an arc of radius $2l$)

Angular Motion





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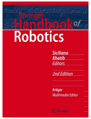

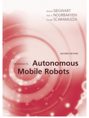

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$$\omega = \frac{r\dot{\varphi}_1 - r\dot{\varphi}_2}{2l}$$

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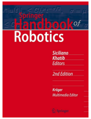

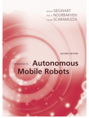

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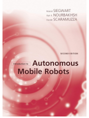
- The distance D of P to the ICR can also be found as

$$D = l\frac{r\dot{\varphi}_1 + r\dot{\varphi}_2}{r\dot{\varphi}_1 - r\dot{\varphi}_2}$$

Forward Kinematics Model



- The complete forward kinematics model of a differential drive platform is given as

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{}^{R}\dot{\boldsymbol{p}} = \begin{pmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{pmatrix}
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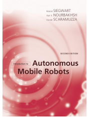

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- With respect to the global reference frame I, the model is given as

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{}^{I}\dot{\boldsymbol{p}} = R(\theta)^{T}\,{}^{R}\dot{\boldsymbol{p}} = R(\theta)^{T} \begin{pmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{pmatrix}
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Motion Constraints

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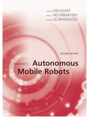
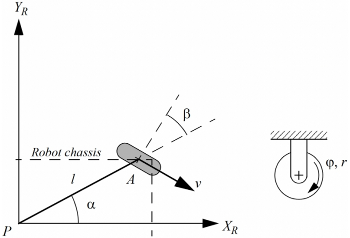
  - **Rolling constraint**, which specifies a wheel's rolling motion (motion parallel to the wheel's plane) with respect to P
  - **Sliding constraint**, which specifies a wheel's sideways sliding motion (motion orthogonal to the wheel's plane) with respect to P

Standard Wheel Constraints





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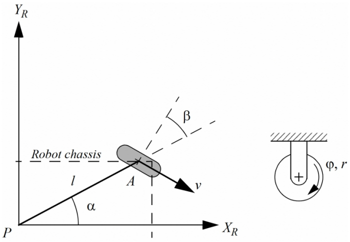

- For a fixed standard wheel, the motion parallel to the wheel's plane is captured by the wheel's rolling, so

Rolling constraint for a fixed standard wheel

$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^{R}_{I}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = r\dot{\varphi}$$

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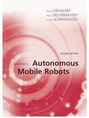

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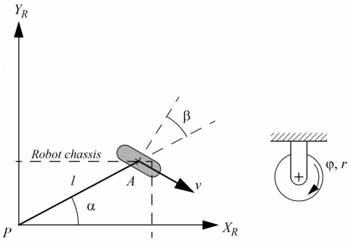

- A standard wheel cannot slide sideways; thus, the velocity orthogonal to the wheel's plane needs to be equal to 0:

Sliding constraint for a fixed standard wheel

$$\begin{pmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin(\beta)\end{pmatrix} {}^{R}_{I}R(\theta)\, {}^{I}\dot{\boldsymbol{p}} = 0$$

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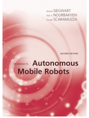

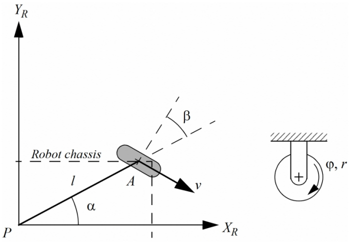

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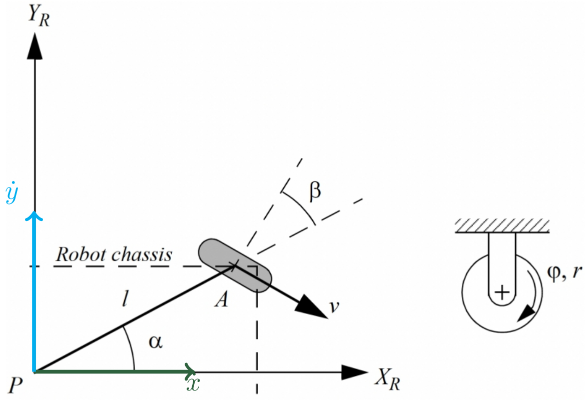
- The sliding and rolling constraint are the same for a steered standard wheel (with a variable β)

Standard Wheel Constraints Derivation



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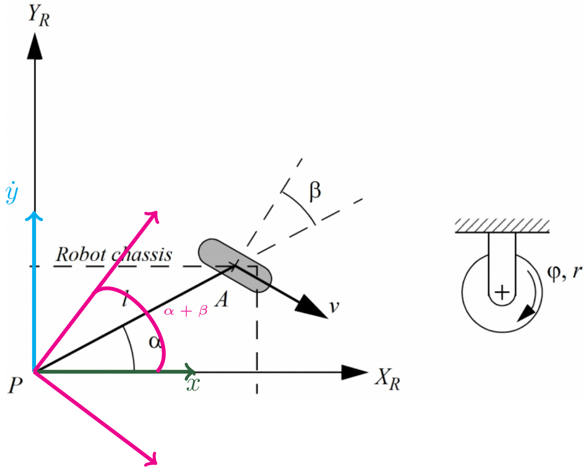

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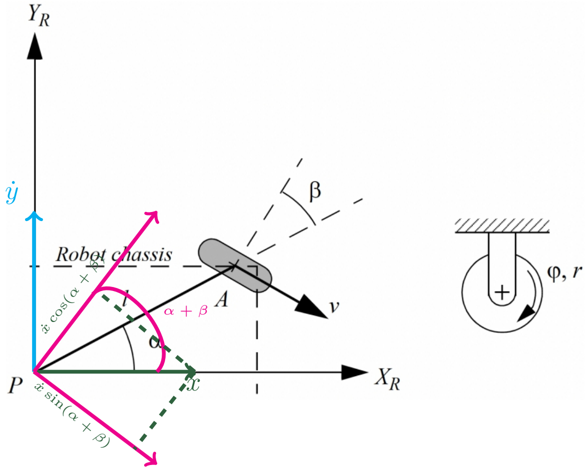

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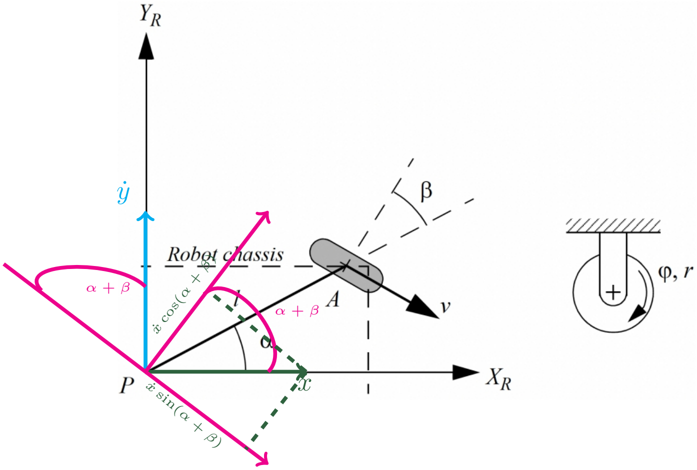

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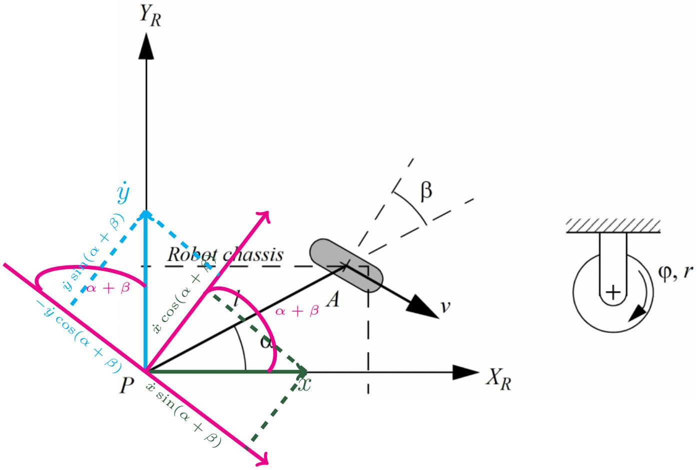

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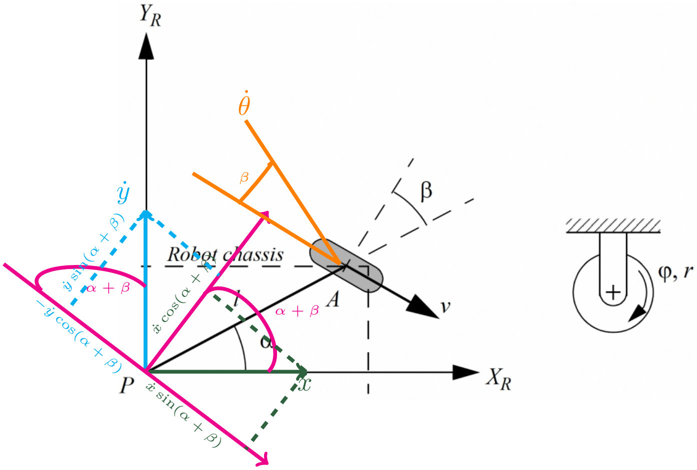

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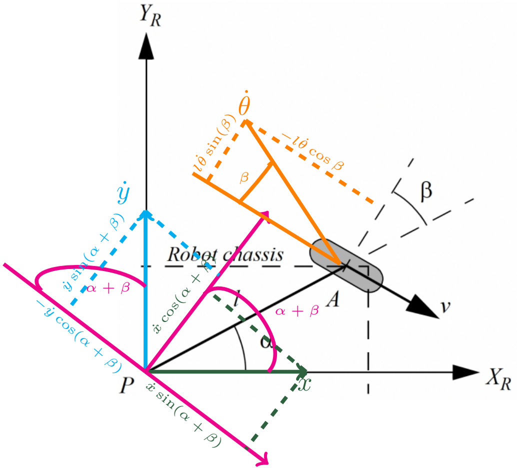
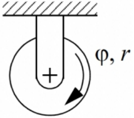
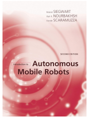
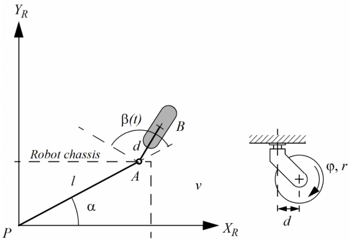


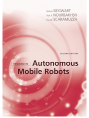


Caster Wheel Constraints



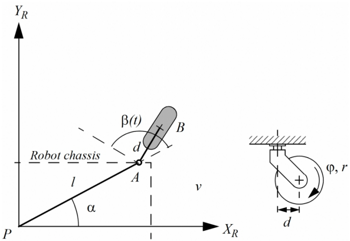


Caster Wheel Constraints



- A caster wheel differs from a steered standard wheel in its vertical rotation axis, which **does not coincide with the wheel's contact point with the ground**

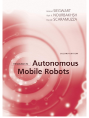
- This offset has no effect on the rolling constraint on the wheel:



Rolling constraint for a caster wheel

$$\left(\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos(\beta)\right) {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

Caster Wheel Constraints



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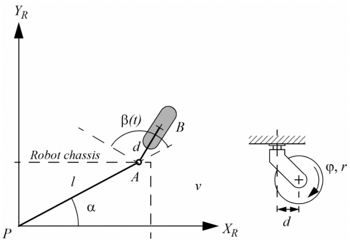


- This offset has no effect on the rolling constraint on the wheel:

Rolling constraint for a caster wheel

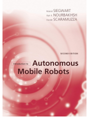
$$\begin{pmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta) \end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

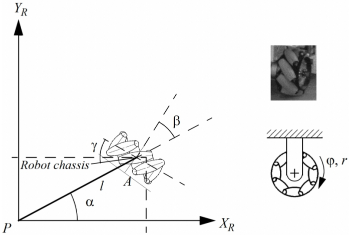
- The offset allows lateral motion, such that a steering motion can counteract any motion orthogonal to the wheel's axis (caused by a force at the caster attachment point A):

Sliding constraint for a caster wheel

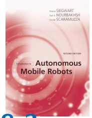
$$\begin{pmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & d + l\sin(\beta) \end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = -d\dot{\beta}$$

Swedish Wheel Constraints

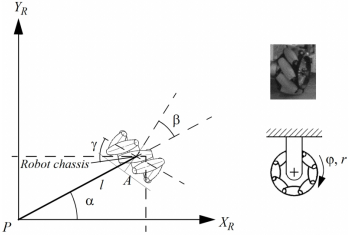




Swedish Wheel Constraints



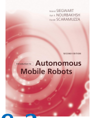
- Unlike a standard wheel, **a Swedish wheel does not have a vertical axis of rotation**
- Its rolling constraint is expressed with respect to an axis at the contact point of the rollers with the ground, which is where rolling occurs:



Rolling constraint for a Swedish wheel

$$\begin{pmatrix}\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l\cos(\beta+\gamma)\end{pmatrix} {}^{R}_{I}R(\theta)\,{}^{I}\dot{\boldsymbol{p}} = r\dot{\varphi}\cos(\gamma)$$

Swedish Wheel Constraints



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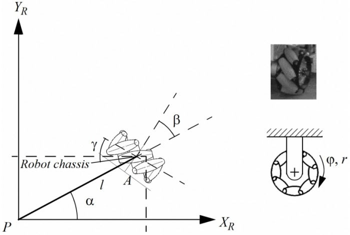


Rolling constraint for a Swedish wheel

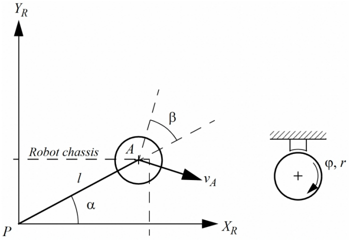
$$\left(\sin(\alpha+\beta+\gamma) \quad -\cos(\alpha+\beta+\gamma) \quad -l\cos(\beta+\gamma)\right) {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}\cos(\gamma)$$

- As rollers can rotate freely with a velocity $\dot{\varphi}_{sw}$, the orthogonal motion with respect to the roller axis is not constrained; this is expressed as

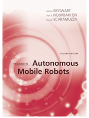
Sliding constraint for a Swedish wheel

$$\left(\cos(\alpha+\beta+\gamma) \quad \sin(\alpha+\beta+\gamma) \quad l\sin(\beta+\gamma)\right) {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}\sin(\gamma) + r_{sw}\dot{\varphi}_{sw}$$

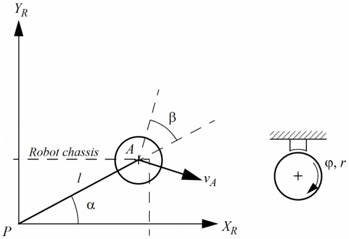
Spherical Wheel Constraints



Spherical Wheel Constraints



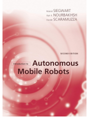
- A spherical wheel has no concrete axis of rotation, so its constraints are defined based on conventions
- The rolling constraint expresses the rolling in the wheel's direction of motion:



Rolling constraint for a spherical wheel

$$\begin{pmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta) \end{pmatrix} {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

Spherical Wheel Constraints



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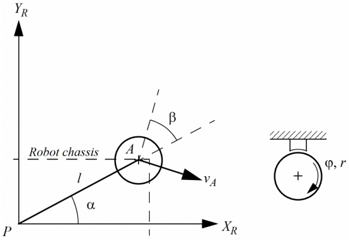


Rolling constraint for a spherical wheel

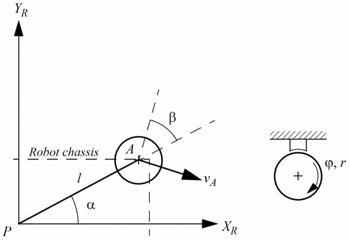
$$\begin{pmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

- The sliding constraint expresses that the wheel does not move orthogonally to the direction of motion:

Sliding constraint for a spherical wheel

$$\begin{pmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin(\beta)\end{pmatrix} {}^R_I R(\theta)\, {}^I\dot{\boldsymbol{p}} = 0$$

Spherical Wheel Constraints



- A spherical wheel has no concrete axis of rotation, so its constraints are defined based on conventions
- The rolling constraint expresses the rolling in the wheel's direction of motion:

Rolling constraint for a spherical wheel

$$\begin{pmatrix} \sin(\alpha + \beta) & -\cos(\alpha + \beta) & -l\cos(\beta) \end{pmatrix} {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = r\dot{\varphi}$$

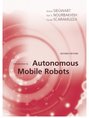
- The sliding constraint expresses that the wheel does not move orthogonally to the direction of motion:

Sliding constraint for a spherical wheel

$$\begin{pmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) & l\sin(\beta) \end{pmatrix} {}^R_I R(\theta) {}^I\dot{\boldsymbol{p}} = 0$$

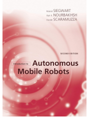
- NB: Mathematically, these are the same as for a standard wheel, but they are interpreted differently because **a spherical wheel does not pose any constraints on a robot's motion**

Kinematic Constraints on a Mobile Base



- To express the kinematic constraints of a mobile robot base, **only fixed and steered standard wheels need to be taken into account** — caster, Swedish, and spherical wheels put no constraints on the motion
- Let us consider **a mobile robot base with $N = N_f + N_s$ standard wheels**, with N_f the number of fixed standard wheels and N_s the number of steered standard wheels
  - We combine the motion of these into a common vector $\dot{\varphi} = (\dot{\varphi}_f \; \dot{\varphi}_s)^T$
- The rolling and sliding constraints for the base are expressed by combining the constraints of the standard wheels

Mobile Base Rolling Constraints



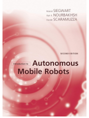
- The rolling constraint of a mobile base is **a system that combines the rolling constraints of the N wheels**, which is expressed as

$$J_1(\beta_s)\,{}^R_I R(\theta)\,{}^I\boldsymbol{p} = J_2\dot{\varphi}$$

- The matrices J_1 and J_2 are given as

$$J_1(\beta_s) = \begin{pmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{pmatrix} \qquad J_2 = \begin{pmatrix} r_{f_1} & & & & & \\ & \ddots & & & & \\ & & r_{f_{N_f}} & & & \\ & & & r_{s_1} & & \\ & & & & \ddots & \\ & & & & & r_{s_{N_s}} \end{pmatrix}$$

Mobile Base Sliding Constraints



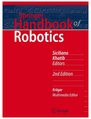
- The sliding constraint is also **a system combining the sliding constraints of the N standard wheels**; this is expressed as follows:

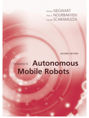
$$C(\beta_s)\,{}_I^R R(\theta)\,{}^I\boldsymbol{p} = \mathbf{0}$$

where

$$C(\beta_s) = \begin{pmatrix} C_f \\ C_s(\beta_s) \end{pmatrix}$$

Instantaneous Center of Rotation (ICR)





- The point around which a mobile robot base rotates is determined by the constraints imposed by its wheels
- This point is the **instantaneous center of rotation (ICR)** that we already encountered for a differential drive robot
- The ICR can be determined by drawing motion lines perpendicularly through the wheel axes; **the intersection point of these lines is the ICR**
  - If the wheels move in a straight line, the ICR is at infinity
- It should be noted that **not all standard wheels provide independent constraints for determining the ICR**
  - For a car, the back wheels are fixed along the same axis, so they do not both independently constrain the vehicle's motion

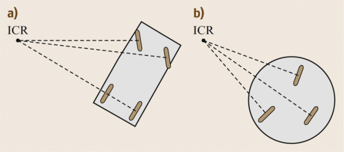
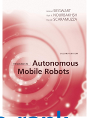


Fig.24.5a,b The instantaneous center of rotation. **(a)** A car-like robot; **(b)** a three-steering-wheels robot

Degree of Mobility

- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\text{rank}(C(\beta_s))$
  - As a wheeled robot moves on a plane, we have that $0 \leq \text{rank}(C(\beta_s)) \leq 3$

Degree of Mobility



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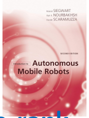
- The **degree of mobility** δ_m of a mobile robot is defined in terms of the null space of $C(\beta_s)$, which is a function of $\mathrm{rank}(C(\beta_s))$:

$$\delta_m = \dim\left(\mathrm{null}\left(C(\beta_s)\right)\right) = 3 - \mathrm{rank}\left(C(\beta_s)\right)$$

Degree of Mobility



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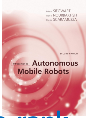
- We can consider the following examples resulting from this definition:
  - A bicycle has $\delta_m = 1$: at any point, it can either move forward or change its rotation

Degree of Mobility



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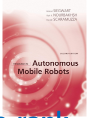
  - A differential drive has $\delta_m = 2$: both the linear speed and orientation can be controlled instantaneously

Degree of Mobility



- **The number of independent sliding constraints on a mobile base is determined by the rank of $C(\beta_s)$**, which we denote by $\text{rank}(C(\beta_s))$
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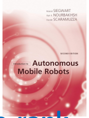
  - A differential drive has $\delta_m = 2$: both the linear speed and orientation can be controlled instantaneously
  - A robot without standard wheels has $\delta_m = 3$

Degree of Mobility



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  - A bicycle has $\delta_m = 1$: at any point, it can either move forward or change its rotation
  - A differential drive has $\delta_m = 2$: both the linear speed and orientation can be controlled instantaneously
  - A robot without standard wheels has $\delta_m = 3$
- The degree of mobility determines the **differentiable degrees of freedom** (DDOF), which is the number of velocities that can be independently achieved by a robot

Degree of Steerability

- The **degree of steerability** δ_s of a mobile robot is defined as the number of steerable parameters that can be independently controlled

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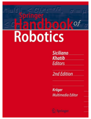
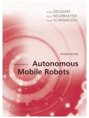
- This also depends on the number of independent constraints imposed by the wheels on the base and is **defined via the rank of** $C_s(\beta_s)$:

$$\delta_s = \text{rank}\left(C_s(\beta_s)\right)$$

Degree of Steerability





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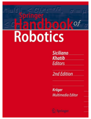
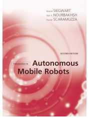
$$\delta_s = \text{rank}\left(C_s(\beta_s)\right)$$

- Steerable wheels decrease the mobility (they impose a sliding constraint), but enable a robot to adapt its overall motion through the steerable orientation

Degree of Steerability





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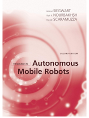
$$\delta_s = \text{rank}\left(C_s(\beta_s)\right)$$

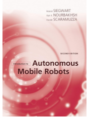
- Steerable wheels decrease the mobility (they impose a sliding constraint), but enable a robot to adapt its overall motion through the steerable orientation
- δ_s is typically defined so that $0 \leq \delta_s \leq 2$; if there are more than two steered wheels, the existence of an ICR may not be guaranteed!

Degree of Maneuverability



- The **degree of maneuverability** δ_M defines the total number of degrees of freedom that can be controlled by a wheeled mobile robot

Degree of Maneuverability



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  - the degrees of freedom that can be controlled directly through the individual wheel velocities and
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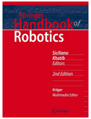
- Intuitively, δ_M summarises:
  - the degrees of freedom that can be controlled directly through the individual wheel velocities and
  - the degrees of freedom whose control is made possible through steering
- δ_M is thus defined as a combination of the degrees of mobility and steerability:

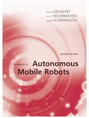
$$\delta_M = \delta_m + \delta_s$$

Basic Types of Wheeled Mobile Robots

Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels

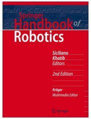
Basic Types of Wheeled Mobile Robots

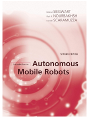




Type	Maneuverability	Description
(3,0)	$\delta_M = 3,\ \delta_m = 3,\ \delta_s = 0$	No fixed or steering wheels
(2,0)	$\delta_M = 2,\ \delta_m = 2,\ \delta_s = 0$	One or more fixed wheels with a common axis; no steering wheels

Basic Types of Wheeled Mobile Robots





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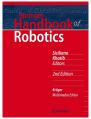
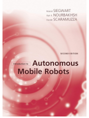
Basic Types of Wheeled Mobile Robots





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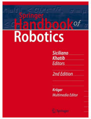

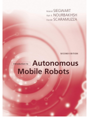

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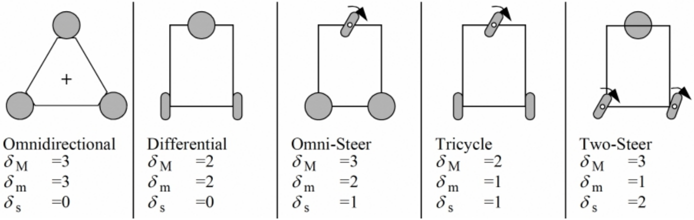
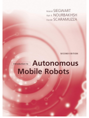


Holonomic and Nonholonomic Constraints



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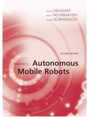

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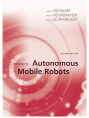

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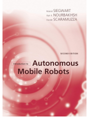

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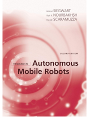
- Nonholonomic constraints are nonintegrable and thus express a dependence on the body's state on the path taken to reach that state

- The sliding constraint of standard wheels is a nonholonomic constraint

Workspace, Paths, Trajectories



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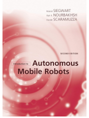

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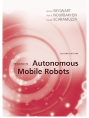

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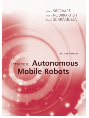

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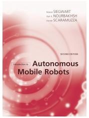
A trajectory combines a path with time, namely it is **a collection of poses along with time instants at which those poses should be reached**

What Does This All Mean in Practice?



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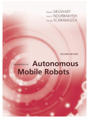

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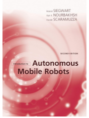

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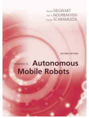

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  - For instance, a car has $\delta_M = 2$, but it is a very useful mobile base (obviously)
- Practically speaking, **mobile bases with varying maneuverabilities differ in the manner in which they are controlled**: a base with lower maneuverability typically requires multiple maneuvers to follow certain paths and may not be able to follow arbitrary trajectories